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Optimization approaches to Economic Dispatching in Electric Energy Systems

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Abstract

This thesis presents a framework for fundamental electrical economy concepts and methodology's. The proposed setup is based on mathematical optimization (nonlinear programming) and the basic tools are: The Lagrangian approach for systems with equality constraints and the Karush Kuhn Tucker (KKT) for optimization with inequality constraints. The proposed design attempts to achieve a trade-off between minimizing the monetary cost and maximizing the power efficiency while meeting with the demand level. The monetary cost objective function consists of the plant's hourly cost function, the power demanded from the load, the transmission power losses and the generators / tie-line power limits.

Two numerical examples are used to show the optimization with both two and three different power plants. Four different cases are being used to show what effect the different constraints have on the demanded power and the different power plants.

Case 1: Optimal Dispatching with equality constraints

Case 2: Optimal Dispatching with equality constraints and power generation limits

Case 3: Optimal Dispatching with equality constraints and transmission losses

Case 4: Optimal Dispatching with equality constraints, generation limits and transmission losses

Introduction

What is optimization:

A broad set of interrelated decisions on obtaining, operating, and maintaining physical and human resources for electricity generation, transmission, and distribution that minimize the total cost of providing electric power to all classes of consumers, subject to engineering, market, and regulatory constraints

Power System Optimization is aimed at improvements in more areas than cost:

- **Reliability:** by reducing the cost of interruptions and power quality disturbances and reducing the probability and consequences of widespread blackouts.
- **Economics:** by keeping downward prices on electricity prices, reducing the amount paid by consumers as compared to the "business as usual" (BAU) grid, creating new jobs and stimulating the economy
- **Efficiency:** by reducing the cost to produce, deliver, and consume electricity.
- **Environmental Friendliness:** by reducing emissions when compared to BAU by enabling a larger penetration of renewables and improving efficiency of generation, delivery, and consumption.
- **Security:** by reducing dependence on imported energy as well as the probability and consequences of manmade attacks and natural disasters.

Chapter 1: Basic economics

1.1 Economic terminology

1.1.1 Elasticity of demand

Increasing the price of a commodity even by a small amount will clearly decrease demand. But by how much? To answer this question, we could use the derivative $\frac{dq}{d\pi}$ of the demand curve. Using this slope directly presents the problem that the numerical value depends on the units that we use to measure the quantity and the price. Comparing the demand's response to price changes for various commodities would be impossible. To get around this difficulty, we define the price elasticity of demand as the ratio of the relative change in demand to the relative change in price:

$$\varepsilon = \frac{\frac{dq}{q}}{\frac{d\pi}{d}} = \frac{\pi}{q}\frac{dq}{d\pi}$$

The demand for a commodity is said to be elastic if a given percentage change in price produces a larger percentage change in demand. On the other hand, if the relative change in demand is smaller than the relative change in price, the demand is said to be inelastic. Finally, if the elasticity is equal to -1, the demand is unit elastic. The elasticity of the demand for a commodity depends in large part on the availability of substitutes. For example, the elasticity of the demand for coffee would be much smaller if consumers did not have the option to drink tea. When discussing elasticities and substitutes, one has to be clear about the timescale for substitutions. Suppose that electric heating is widespread in a region. In the short run, the price elasticity of the demand for electricity is very low because consumers do not have a choice if they want to stay warm. In the long run, however, they can install gas-fired heating and the price elasticity of the demand for electricity will be much higher. The concept of substitute products can be quantified by defining the cross-elasticity between the demand for commodity I and the price of commodity j:

$$\varepsilon_{ij} = \frac{\frac{dq_i}{q_i}}{\frac{d\pi_j}{d_j}} = \frac{\pi_j}{q_i}\frac{dq_i}{d\pi_j}$$

While the elasticity of a commodity to its own price (its self-elasticity) is always negative, cross-elasticities between substitute products are positive because an increase in the price of one will spur the demand for the other. If two commodities are complements, a change in the demand for one will be accompanied by a similar change in the demand for the other. Electricity and electric heaters are clearly complements. The cross-elasticities of complementary commodities are negative.

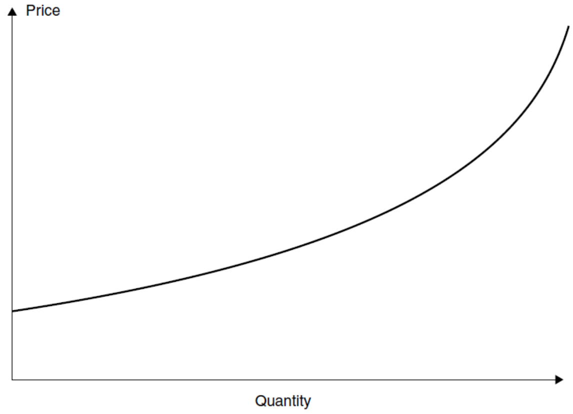
1.1.2 Opportunity cost

Opportunity cost is the utility or profits foregone by choosing one alternative over another. That means that consumers have the choice of how much they purchase from a commodity. So they can decide if the price they pay is on the same marginal level as the benefit they get. The producer can make different decisions based on what revenue he will get from selling at a specific market. He has the choice to sell his commodity directly to the consumer, sell it on the market or can stop the production. We can say that the sale of the commodity is less than the opportunity cost associated with the production of it.

Our model of the consumers' behaviour is based on the assumption that these consumers can choose how much of a commodity they purchase. We also argued that the consumption level is such that the marginal benefit that consumers get from this commodity is equal to the price that they have to pay to obtain it. A similar argument can be used to develop our model of the producers. Let us consider one of the apple growers who brings her products to the market that we visited earlier. There is a price below which she will decide that selling apples is not worthwhile. There are several reasons why she could conclude that this revenue is insufficient. First, it might be less than the cost of producing the apples. Second, it might be less than the revenue she could get by using these apples for some other purposes, such as selling them to a cider-making factory. Finally, she could decide that she would rather devote the resources needed to produce apples (money, land, machinery and her own time) into some other activity, such as growing pears or opening a bed-and-breakfast. One can summarize these possibilities by saying that the revenue from the sale of apples is less than the opportunity cost associated with the production of these apples.

1.1.3 Supply and inverse supply

While the elasticity of a commodity to its own price (its self-elasticity) is always negative, cross-elasticities between substitute products are positive because an increase in the price of one will spur the demand for the other. If two commodities are complements, a change in the demand for one will be accompanied by a similar change in the demand for the other. Electricity and electric heaters are clearly complements. The cross-elasticities of complementary commodities are negative. Other producers have different opportunity costs and will therefore decide to adjust the amount they supply at different price thresholds. If we aggregate the amounts supplied by a sufficiently large number of producers, we get a smooth, upward-sloping curve such as the one shown in adjacent graph this curve represents the inverse supply function for this commodity:

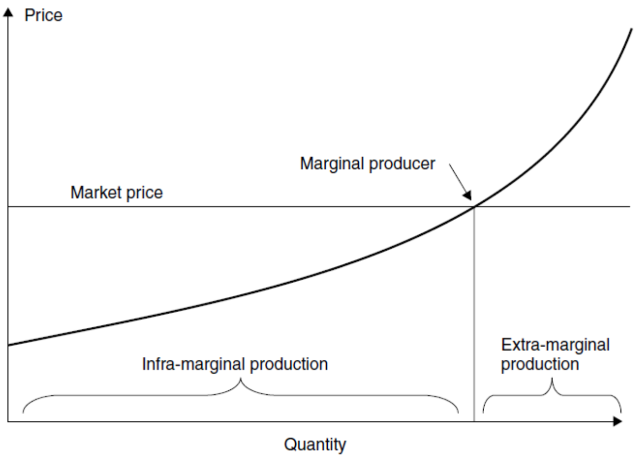


$$\pi = S^{-1}(q)$$

This function indicates the value that the market price should take to make it worthwhile for the aggregated producers to supply a certain quantity of the commodity to the market. We can, of course, look at the same curve from the other direction and define the supply function, which gives us the quantity supplied as a function of the market price:

$$q = S(\pi)$$

As depicted in adjacent graph, goods produced by different producers (or by the same producer but using different means of production) are located on different parts of the supply curve. The marginal producer is the producer whose opportunity cost is equal to the market price. If this market



price decreases even by a small amount, this producer would decide that it is not worthwhile to continue production. Extra marginal production refers to production that could become worthwhile if the market price were to increase. On the other hand, the opportunity cost of the infra-marginal producers is below the market price. These producers are thus able to sell at a price that is higher than the lowest price at which they would find it worthwhile to produce.

1.1.4 Market equilibrium

So far, we have considered producers and consumers separately. It is time to see how they interact in a market. In this section, we make the assumption that each supplier or consumer cannot affect the price by its actions. In other words, all market participants take the price as given. If this assumption is true, the market is said to be a perfectly competitive market. This assumption is usually not true for electricity markets. We will thus discuss in a later section how markets operate when some participants can influence the price through their actions. In a competitive market, it is the combined action of all the consumers on one side and of all the suppliers on the other side that determines the price. The equilibrium price or market clearing price

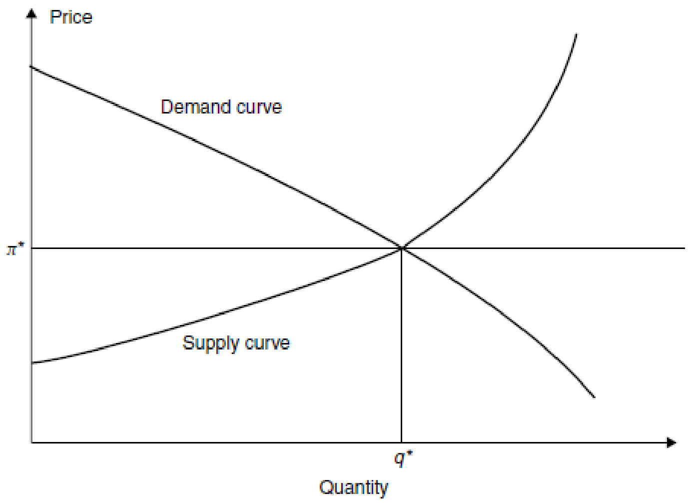
$*$ is such that the quantity that the suppliers are willing to provide is equal to the quantity that the consumers wish to obtain. It is thus the solution of the following equation:

$$D(\pi *) = S(\pi *)$$

This equilibrium can also be defined in terms of the inverse demand function and the inverse supply function. The equilibrium quantity q$*$ is such that the price that the consumers are willing to pay for that quantity is equal to the price that producers must receive to supply that quantity:

$$D^{-1}(q *) = S^{-1}(q *)$$

The graph below illustrates these concepts.



So far, we have shown that at the market equilibrium, the behaviours of the consumers and the suppliers are consistent. We have not yet shown, however, that this point represents a stable equilibrium. To demonstrate this, let us show that the market will inevitably settle at that point. Suppose, as shown in Figure 2.13, that the market price is

, where the demand is greater than the supply. Some suppliers will inevitably realize that there are some unsatisfied customers to whom they could sell their goods at more than the going price. The traded quantity will increase and so will the price until the equilibrium conditions are reached.

Similarly, if the market price is п2 > п ∗, the supply exceeds the demand and some suppliers are left with goods for which they cannot find buyers. To avoid being caught in this situation, they will reduce their production until the amount that producers are willing to sell is equal to the amount that consumers are willing to buy.

1.1.5 Pareto efficiency

When the market is controlled by only 1 supplier, the supplier will try to increase the benefit it makes from it. If the market is depends on different suppliers, than it is not possible to increase its benefit of 1 supplier without reducing the benefit of the other suppliers. This is called pareto efficiency.

Pareto efficiency, or Pareto optimality, is a state of allocation of resources in which it is impossible to make any one individual better off without making at least one individual worse off. The term is named after Vilfredo Pareto (1848–1923), an Italian engineer and economist who used the concept in his studies of economic efficiency and income distribution. The concept has applications in academic fields such as economics, engineering, and the life sciences.

Pareto improvement is defined to be a change to a different allocation that makes at least one individual better off without making any other individual worse off, given a certain initial allocation of goods among a set of individuals. An allocation is defined as "Pareto efficient" or "Pareto optimal" when no further Pareto improvements can be made.

Pareto efficiency is a minimal notion of efficiency and does not necessarily result in a socially desirable distribution of resources: it makes no statement about equality, or the overall well-being of a society. The notion of Pareto efficiency can also be applied to the selection of alternatives in engineering and similar fields. Each option is first assessed under multiple criteria and then a subset of options is identified with the property that no other option can categorically outperform any of its members

If economic allocation in any system is not Pareto efficient, there is potential for a Pareto improvement—an increase in Pareto efficiency: through reallocation, improvements can be made to at least one participant's well-being without reducing any other participant's well-being.

It is important to note, however, that a change from an inefficient allocation to an efficient one is not necessarily a Pareto improvement. Thus, in practice, ensuring that nobody is disadvantaged by a change aimed at achieving Pareto efficiency may require compensation of one or more parties. For instance, if a change in economic policy eliminates a monopoly and that market subsequently becomes competitive and efficient, the monopolist will be made worse off. However, the loss to the monopolist will be more than offset by the gain in efficiency, in the sense that the monopolist could hypothetically be compensated for its loss while still leaving a net gain for others in the economy, a Pareto improvement.

In real-world practice, such compensations have unintended consequences. They can lead to incentive distortions over time as agents anticipate such compensations and change their actions accordingly. Under certain idealized conditions, it can be shown that a system of free markets will lead to a Pareto efficient outcome. This is called the first welfare theorem. It was first demonstrated mathematically by economists Kenneth Arrow and Gérard Debreu. However, the result only holds under the restrictive assumptions necessary for the proof (markets exist for all possible goods so there are no externalities, all markets are in full equilibrium, markets are perfectly competitive, transaction costs are negligible, and market participants have perfect information). In the absence of perfect information or complete markets, outcomes will generically be Pareto inefficient, per the Greenwald-Stiglitz theorem.

1.2 Discussion about energy exchange

1.2.1 Spot market

In a spot market, the seller delivers the goods immediately and the buyer pays for them "on the spot". No conditions are attached to the delivery. This means that neither party can back out of the deal. A fruit and vegetable market is a good example of a spot market: you inspect the quality of the produce and tell the vendor how many cucumbers you want, she hands them to you, you pay the price indicated and the transaction is complete. If later on you decide that you would rather eat lettuce, you probably would not even think of trying to return the cucumbers and getting your money back. On the surface, the rules of such markets may appear very informal. In fact, they have behind them the weight of centuries of tradition. Modern spot markets for commodities such as oil, coffee or barley are superficially more sophisticated because the quantities traded are much larger and because traders communicate electronically. However, the principles are exactly the same.

A spot market has the advantage of immediacy. As a producer, I can sell exactly the amount that I have available. As a consumer, I can purchase exactly the amount I need. Unfortunately, prices in a spot market tend to change quickly. A sudden increase in demand (or a drop in production) sends the price soaring because the stock of goods available for immediate delivery may be limited. Similarly, a glut in production or a dip in demand depresses the price. Spot markets also react to news about the future availability of a commodity. For example, a forecast about a bumper harvest of an agricultural commodity could send its price on the spot market (the spot price) Electricity pool plunging if enough consumers have the ability to wait until this harvest comes to market. Changes in the spot price are essentially unpredictable because if they were predictable, the market participants would anticipate them. Large and unpredictable variations in the price of a commodity make life harder for both suppliers and consumers of this commodity. Both are running businesses and are thus facing a variety of risks. Bad weather or a pest can ruin a harvest. The breakdown of a machine can stop production. A strike can stop the shipment of finished goods. While being in business means taking some risks, an excessive amount of risk endangers the survival of a business. Most businesses will therefore try to reduce their exposure to price risks. For example, the producer of a commodity will try to avoid being forced to sell its output at a very low price. Similarly, a consumer does not want to be obliged to buy an essential commodity at a very high price. This desire to avoid being exposed to the wild price fluctuations that are common in spot markets has led to the introduction of other types of transactions and markets. These markets are described in the following sections.

1.2.2 Bilateral trading

As its name implies, bilateral trading involves only two parties: a buyer and a seller. Participants thus enter into contracts without involvement, interference or facilitation from a third party. Depending on the amount of time available and the quantities to be traded, buyers and sellers will resort to different forms of bilateral trading:

- *Customized long-term contracts* the terms of such contracts are flexible since they are negotiated privately to meet the needs and objectives of both parties. They usually involve the sale of large amounts of power (hundreds or thousands of MW) over long periods of time (several months to several years). The large transaction costs associated with the negotiation of such contracts make them worthwhile only when the parties want to buy or sell large amounts of energy.

- *Trading "over the counter"* These transactions involve smaller amounts of energy to be delivered according to a standard profile, that is, a standardized definition of how much energy should be delivered during different periods of the day and week. This form of trading has much lower transaction costs and is used by producers and consumers to refine their position as delivery time approaches.

•*Electronic trading* Participants can enter offers to buy energy and bids to sell energy directly in a computerized marketplace. All market participants can observe the quantities and prices submitted but do not know the identity of the party that submitted each bid or offer. When a party enters a new bid, the software that runs the exchange checks to see if there is a matching offer for the period of delivery of the bid. If it finds an offer whose price is greater than or equal to the price of the bid, a deal is automatically struck and the price and quantity are displayed for all participants to see. If no match is found, the new bid is added to the list of outstanding bids and will remain there until a matching offer is made or the bid is withdrawn or it lapses because the market closes for that period. A similar procedure is used each time a new offer is entered in the system. This form of trading is extremely fast and cheap. A flurry of trading activity often takes place in the minutes and seconds before the closing of the market as generators and retailers fine-tune their position ahead of the delivery period.

The essential characteristic of these three forms of bilateral trading is that the price of each transaction is set independently by the parties involved. There is thus no "official" price. While the details of negotiated long-term contracts are usually kept private, some independent reporting services usually gather information about over the counter trading and publish summary information about prices and quantities in Managed spot market a form that does not reveal the identity of the parties involved. This type of market reporting and the display of the last transaction arranged through electronic trading enhance the efficiency of the market by giving all participants a clearer idea of the state and the direction of the market.

1.2.3 Electricity pools

In the early days of the introduction of competition in electrical energy trading, bilateral trading was seen as too big a departure from the existing practice. Since electrical energy is pooled as it flows from the generators to the loads, it was felt that trading might as well be done in a centralized manner and involve all producers and consumers. Competitive electricity pools were thus created. Pools are a very unusual form of commodity trading but they have well-established roots in the operation of large power systems. In fact, some of the competitive electricity pools currently in operation were developed on the basis of collaborative pools created by monopoly utility companies

With adjacent service territories. Rather than relying on repeated interactions between suppliers and consumers to reach the market equilibrium, a pool provides a mechanism for determining this equilibrium in a systematic way. While there are many possible variations, a pool essentially operates as follows:

- Generating companies submit bids to supply a certain amount of electrical energy at a certain price for the period under consideration. These bids are ranked in order of increasing price. From this ranking, a curve showing the bid price as a function of the cumulative bid quantity can be built. This curve is deemed to be the supply curve of the market.

- Similarly, the demand curve of the market can be established by asking consumers to submit offers specifying quantity and price and ranking these offers in decreasing order of price. Since the demand for electricity is highly inelastic, this step is sometimes omitted and the demand is set at a value determined using a forecast of the load. In other words, the demand curve is assumed to be a vertical line at the value of the load forecast.

- The intersection of these "constructed" supply and demand curves represents the market equilibrium. All the bids submitted at a price lower than or equal to the market clearing price are accepted and generators are instructed to produce the amount of energy corresponding to their accepted bids. Similarly, all the offers submitted at a price greater than or equal to the market clearing price are accepted and the consumers are informed of the amount of energy that they are allowed to draw from the system.

- The market clearing price represents the price of one additional megawatt-hour of energy and is therefore called the system marginal price or SMP. Generators are paid this SMP for every megawatt-hour that they produce, whereas consumers pay the SMP for every megawatt-hour that they consume, irrespective of the bids and offers that they submitted.

Paying the SMP for all the generation that was accepted may appear surprising at first glance. Why shouldn't generators that were willing to produce for less be paid only their asking price? Wouldn't this approach reduce the average price of electricity? The main reason this pay-as-bid scheme is not adopted is that it would discourage generators from submitting bids that reflect their marginal cost of production. All generators would instead try to guess what the SMP is likely to be and would bid at that level to collect the maximum revenues. At best, the SMP would therefore remain unchanged. Inevitably, some low-cost generators would occasionally overestimate the value of SMP and bid too high. These generators would then be left out of the schedule and be replaced by generators with a higher marginal cost of production. The SMP would then be somewhat higher than it ought to be. Furthermore, this substitution is economically inefficient because optimal use is not made of the available resources. In addition, generators are likely to increase their prices slightly to compensate themselves for the additional risk of losing revenue because of the uncertainty on the SMP. An attempt to reduce the price of electricity would therefore result in a price increase!

1.2.4 Gate closure

As we argued above, energy trading must stop at some point before real time to give the SO time to balance the system. How much time should elapse between this gate closure and real time is a hotly debated issue. System operators prefer longer intervals because this gives them more time to develop their plans and more flexibility in their selection of balancing resources. For example, if the gate closes half an hour before real time, there is not enough time to bring on-line a large coal-fired plant to make up a deficit in generation.

Participants in the energy market, on the other hand, usually prefer a shorter gate closure because it reduces their exposure to risk. A load forecast calculated one hour ahead of real time is usually much more accurate than a forecast calculated four hours ahead. A retailer would therefore like to trade electronically up to the last minute to match its purchases with its anticipated load. This is considered preferable to relying on the managed spot market in which it is exposed to prices over which it has no control. Generators too prefer shorter gate closures because of the risk of sudden unit outage. If a unit fails after gate closure, there is nothing that the generator can do except hope that the spot market price will not be too high. On the other hand, if the unit fails before gate closure, the generator can try to make up the deficit in generation by purchasing at the best possible price on the electronic exchange. In general, traders prefer a true spot market that is driven solely by market forces to a managed spot market that is heavily influenced by complex technical considerations.

1.2.5 Settlement process

Commercial transactions are normally settled directly between the two parties involved: following the delivery of the goods by the seller to the buyer, the buyer pays the seller the agreed price. If the amount delivered is less than the amount contracted, the buyer is entitled to withhold part of the payment. Similarly, if the buyer consumes more than the agreed amount, the seller is entitled to an additional payment. This process is more complex for electricity markets because the energy is pooled during its transmission from the producers to the consumers. This is why a centralized settlement system is needed.

For bilateral transactions in electrical energy, the buyer pays the seller the agreed price as if the agreed quantity had been delivered exactly. Similarly, the anonymous transactions arranged through screen-based trading are settled through the intermediary of the power exchange as if they had been executed perfectly. However, there will always be inaccuracies in the completion of the contracts. If a generator fails to produce the amount of energy that it has contracted to sell, the deficit cannot simply be withheld from this generator’s customers. Instead, to maintain the stability of the system, the system operator buys replacement energy on the managed spot market.

Similarly, if a large user or retailer consumes less than it has bought, the system operator sells the excess on the managed spot market. These balancing activities make all bilateral contracts look as if they have been fulfilled perfectly. They also carry a cost. In most cases, the amount of money paid by the system operator to purchase replacement energy is not equal to the amount of money earned when selling excess energy. The parties that are responsible for the imbalances should pay the cost of these balancing activities.

The first step in the settlement process consists, therefore, in determining the net position of every market participant. To this end, each generator must report to the settlement system the net amount of energy that it had contracted to sell for each period, including the energy traded through the managed spot market. This amount is subtracted from the amount of energy that it actually produced. If the result is positive, the generator is deemed to have sold this excess energy to the system. On the other hand, if the result is negative, the generator is treated as if it had bought the difference from the system.

Similarly, all large consumers and retailers must report the net amount of energy that they had contracted to buy for each period, including the energy traded through the managed spot market. This amount is subtracted from the amount of energy actually consumed. Depending on the sign of the result, the consumer or the retailer is deemed to have sold energy to the system or bought energy from the system. These imbalances are charged at the spot market price. If this market is suitably competitive, this price should reflect the incremental cost of balancing energy. It is debatable whether the cost of the energy supplied by participants providing ancillary services should be included in this price. Settlement in a pool-based electricity market is more straightforward because all transactions take place through the pool.

1.3 Barrel of oil vs kWh

A barrel of oil equivalent (BOE) is a term used to summarize the amount of energy that is equivalent to the amount of energy found in a barrel of crude oil. There 159 liters in one barrel of oil, which will contain approximately 5.8 million British Thermal Units (MBtus) or 1,700 kilowatt hours (kWh).

The amount of fuel used to generate electricity depends on the efficiency or heat rate of the generator (or power plant) and the heat content of the fuel. Power plant efficiencies (heat rates) vary by types of generators, power plant emission controls, and other factors. Fuel heat contents also vary.

Heat contents vary by type of petroleum product. The development of electricity markets is based on the premise that electrical energy can be treated as a commodity. There are, however, important differences between electrical energy and other commodities such as bushels of wheat, barrels of oil or even cubic meters of gas. These differences have a profound effect on the organization and the rules of electricity markets. The most fundamental difference is that electrical energy is inextricably linked with a physical system that functions much faster than any market. In this physical power system, supply and demand – generation and load – must be balanced on a second-by-second basis. If this balance is not maintained, the system collapses with catastrophic consequences. Such a breakdown is intolerable because it is not only the trading system that stops working but also an entire region or country that may be without power for many hours. Restoring a power system to normal operation following a complete collapse is a very complex process that may take 24 h or more in large, industrialized countries. The social and economic consequences of such a system wide blackout are so severe that no sensible government would agree to the implementation of a market mechanism that significantly increases the likelihood of such an event. Balancing the supply and the demand for electrical energy in the short run is thus a process that simply cannot be left to a relatively slow-moving and unaccountable entity such as a market. In the short run, this balance must be maintained, at practically any cost, through a mechanism that does not rely on a market to select and dispatch resources. Another significant (but somewhat less fundamental) difference between electrical energy and other commodities is that the energy produced by one generator cannot be directed to a specific consumer. Conversely, a consumer cannot take energy from only one generator. Instead, the power produced by all generators is pooled on its way to the loads. This pooling is possible because units of electrical energy produced by different generators are indistinguishable. Pooling is desirable because it results in valuable economies of scale: the maximum generation capacity must be commensurate with the maximum aggregated demand rather than with the sum of the maximum individual demands. On the other hand, a breakdown in a system in which the commodity is pooled affects everybody, not just the parties to a particular transaction. Finally, the demand for electrical energy exhibits predictable daily and weekly cyclical variations. However, it is by no means the only commodity for which the demand is cyclical. The consumption of coffee, to take a simple example, exhibits two or three rather sharp peaks every day, separated by periods of lower demand. Trading in coffee does not require special mechanisms because consumers can easily store it in solid or liquid form. On the other hand, electrical energy must be produced at the same time as it is consumed. Since its short-run price elasticity of demand is extremely small, matching supply and demand requires production facilities capable of following the large and rapid changes in consumption that take place over the course of a day. Not all of these generating units will be producing throughout the day. When the demand is low, only the most efficient units are likely to be competitive and the others will be shut down temporarily. These less efficient units are needed only to supply the peak demand. Since the marginal producer changes as the load increases and decreases, we should expect the marginal cost of producing electrical energy (and hence the spot price of this energy) to vary over the course of the day. Such rapid cyclical variations in the cost and price of a commodity are very unusual. One could argue that trading in gas also takes place over a physical network in which the commodity is pooled and the demand is cyclical. However, the amount of energy stored in the gas pipelines is considerably larger than the amount of kinetic energy stored in electricity-generating units. An imbalance between production and consumption of gas would therefore have to last much longer before it would cause a collapse of the pipeline network. Unlike an imbalance in a power system, it can be corrected through a market mechanism.

1.4 Cost

In this section, we define the various components of the production cost and introduce various curves that are used to characterize these costs. In the short run, some factors of production are fixed. The cost associated with these factors does not depend on the amount produced and is thus a fixed cost. For example, if a generating company has bought land and built a power plant on this land, the costs of the land and the plant do not depend on the amount of energy that this plant produces. On the other hand, the quantity of fuel consumed by this plant and, to a certain extent, the manpower required to operate it depend on the amount of energy it produces. Fuel and manpower costs are thus examples of variable costs. There is also a third class of costs called quasi-fixed costs. These are costs that the firm incurs if the plant produces any amount of output but does not incur if the plant produces nothing. For example, in the case of a generating plant, the cost of the fuel required to start up the plant is fixed in the sense that it does not depend on the amount of energy that the plant goes on producing. However, this start-up cost does not need to be paid if the plant stays idle. In the long run, there are no fixed costs because the firm can decide on the amount of money it spends on all production factors. At the limit, the firm's long-run costs can be zero if it decides to produce nothing and goes out of business. A sunk cost is the difference between the amount of money a firm pays for a production factor and the amount of money it would get back if it sold this asset. For example, in the case of a power plant, the cost of the land on which the plant is built is not a sunk cost because land can always be resold. It is thus a recoverable cost. On the other hand, if production with this plant is no longer profitable, the difference between the cost of building the plant and its scrap metal value is a sunk cost.

1.4.1 Short run costs

The short run cost is important to consider in the electricity market. This is mostly because the start-up cost of a power plant is high. If a coal power plant would only run for 1 hour, the short run cost is very high. If it would run for a couple of days, the short run cost is much lower.

It depends on which sort of power plant it is. In case of a hydro plant, the start-up cost is equal to 0 as is a nuclear plant. With a coal plant the cost is very high.

A nuclear power plant's start-up cost is equal to 0 because the operating and fuel costs are measured in units of $/MWh suggesting that these operating costs and fuel costs are variable or even marginal. However, these numbers are calculated from fixed costs divided by unit output over some period (e.g., a year). Nuclear operating and fuel costs calculated this way increase if output is lower, a strong indication that the costs are not variable or marginal.

Calculating nuclear fuel costs in units of $/MWh allows comparisons to combustion-based generation fuel costs, but this approach is not useful in understanding nuclear power short-run cost.

Nuclear fuel costs are incurred well before nuclear fuel is loaded during a refuelling outage. Nuclear fuel costs are not changed by a small and temporary change in output during the operating period that follows the refuelling outage.

Regulated utility ratemaking estimates nuclear fuel cost, in $/MWh, by dividing the cost of nuclear fuel for a coming operating period by the plant's projected output for the future operating period. This regulatory estimate of nuclear fuel cost is used to recover the cost of nuclear fuel in regulated rates for the future operating period. Regulatory recovery of nuclear fuel cost includes a "true-up" process after the operating period. The money collected in rates for nuclear fuel may be more or less than the actual nuclear fuel cost due to differences between actual and projected total output over the operating period. The true-up amounts are used to adjust rates in the future.

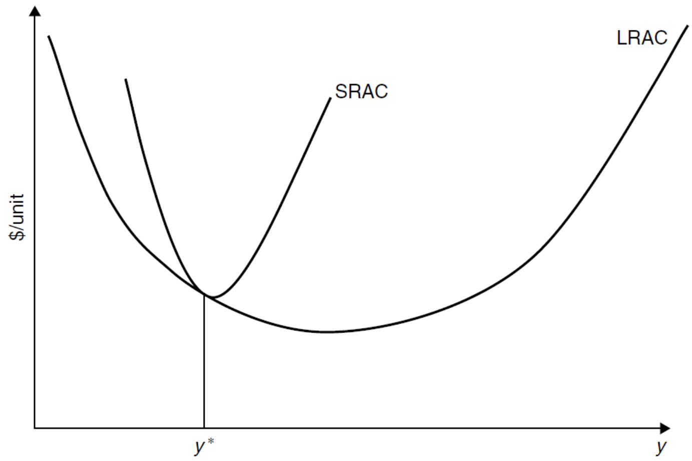
A long nuclear plant outage might delay a scheduled nuclear refuelling outage, but a small change in output for a short period would not. PWR and BWR nuclear power plants shut down and conduct a refuelling/maintenance outage every 18 to 24 months. A nuclear power plant refuelling outage is a period of intense and highly coordinated maintenance activity that is only partly related to the actual refuelling activity.

A small change in output for a short period will not change the refuelling outage schedule or the fuel costs.

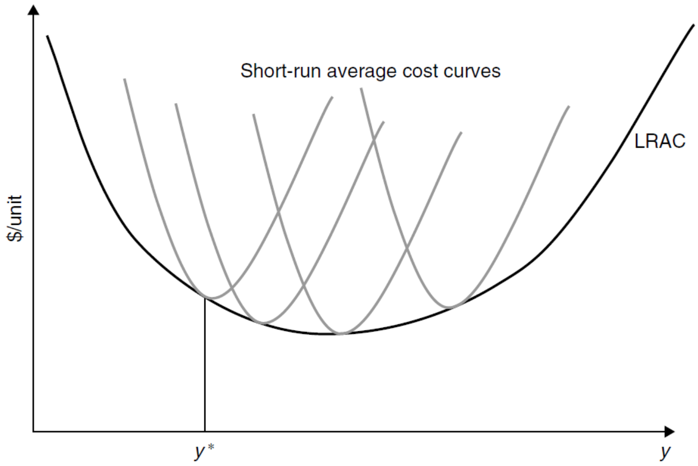
1.4.2 Long run costs

We have argued above that, in the long run, there are no fixed costs because all the factors of production can be changed and the firm has the option to produce nothing and get out of business. However, the technology may be such that some costs are incurred independently of the level of production. There may therefore be some quasi fixed costs in the long run. The long-run average cost curve therefore tends to have a U-shape, as shown in the Figure below.

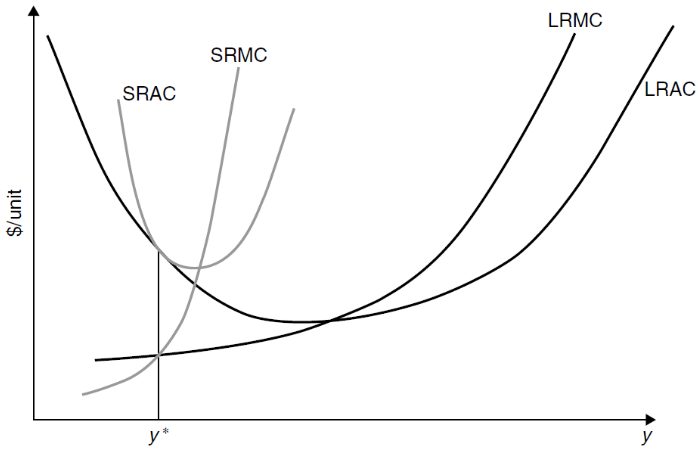
What can we say about the relation between the short-run cost and the long-run cost? In the long run, we can minimize the production cost for any level of output because we can adjust all the factors of production. On the other hand, in the short run, some of the production factors are fixed. The short-run Fixed costs production cost is therefore equal to the long-run production cost only for the value of output y* for which the fixed production factors were optimized. For other levels of output, the short-run cost is higher than the long-run cost. The short-run average cost curve is therefore above the long-run average cost curve, except for the output for which the fixed production factors have been optimized. At that point, the two curves are tangent, as shown in the figure on the side.



We could, of course, select other sets of fixed production factors that would minimize the production cost for other values of the output y1, y2, . . . yn. In other words, we could build plants with other capacities. For each plant size, the short-run average cost would be equal to the long-run average cost only for the designed plant capacity. As the figure below shows, the long-run average cost curve is therefore the lower envelope of the short-run average cost curves.



When all factors of production can be adjusted, the cost of a unit increase in production is given by the long-run marginal cost curve. The figure below illustrates two observations about this long-run marginal cost curve. First, the long-run and short-run marginal costs are equal only for the production level y* for which the fixed production factors have been optimized. Second, the long-run marginal cost is equal to the long-run average cost for the production level that results in the minimum long-run average cost. As long as the long-run marginal cost is smaller than the long-run average cost, this long-run average cost decreases. As long as the average cost decreases, the production is said to exhibit economies of scale



Long run marginal cost is defined as the marginal cost of supplying an additional unit of electrical energy when the installed capacity of the system, under specified reliability standards, is allowed to increase optimally in response to the marginal increase in demand. As such, it incorporates both capital and operating costs. The value of LRMC reflects the marginal cost of optimal production capacity expansion (forward-looking model) required to support a marginal increase in demand within a pre-defined planning horizon.

Chapter 2: Fundamental issues in power economics

2.3 hydrothermal coordination / scheduling

- Different from other plants
- No 2 are alike
- Problems:
    - When building, large areas are flooded
    - Less water after the plant -> no water for irrigation
    - Security
    - Water level too low for boats
- Constraints:
    - Build before other plants -> minimum amount of water
    - Fish in reservoir

Too much water release -> big wave

Scheduling

- Up to years of scheduling
- Water used at a rate of filling the reservoir
    - Depends on weather, melting of snow
    - Problems
    - Hard to meet all the constraints

Scheduling energy

- 2 generating units: Hydro and steam
- Minimize use of steam unit
- Steam plant used at minimum and at constant cost
- Hydro plant changes its output, steam plant at constant output
- Hydro plant is limited to certain amount of time

Pumped storage hydro plant

- Used when peak loads to save fuel costs
- Pumping water back up = saving energy
- Pumping water to fill the reservoir for use later

Dynamic programming

- Use of a special algorithm to schedule the hydro unit
- Considers the variables
  - Inflow rate
  - Volume of storage
  - Flow rate
  - Power output
  - Spillage
  - Hydraulic coupling (plants build after each other)

2.4 Unit commitment issue

Economic Dispatch: Problem Definition

- Given load
- Given set of units on-line
- How much should each unit generate to meet this load at minimum cost?

Constraints

- Unit constraints: Maximum / minimum generating capacity
  Minimum "up time"
  Minimum "down time"
- System constraints: Load/generation balance
  Reserve generation capacity
  Emission constraints
  Network constraints
  - Start-up costs
  - Cost incurred when we start a generating unit
  - Different units have different start-up costs

Flexible plants

- Power output can be adjusted (within limits)
- Examples:
  - Coal-fired
  - Oil-fired
  - Open cycle gas turbines
  - Combined cycle gas turbines
  - Hydro plants with storage
- Status and power output can be optimized

Inflexible plants

- Power output cannot be adjusted for technical or commercial reasons
- Examples:
  - Nuclear
  - Run-of-the-river hydro
  - Renewables (wind, solar,…)
- Output treated as given when optimizing

Unit commitment

- Total generation = Demand + reserve
- Reserve: Protects frequency if loss of unit
  Rapid increase demand
- Using unit commitment to save money
- 3 methods Priority list method
  Dynamic programming
  Lagrange optimization

Priority list method

- Divide units according to priority
  - Cost effective -> least cost effective
- Using cost effective plant the most
- Change in load, follow these steps:
  - Shutting down, enough generation demand + reserve
  - Hours of shutdown

    - Less / more than minimum shutdown time
    - Calculate : Cost plant running at minimum
      Cost plant shutting off + start up (banking or cooling)
    - Repeat

Dynamic programming

- Divide units according to priority
    - Cost effective -> least cost effective
- Using cost effective plant the most
- Change in load, follow these steps:
    - Shutting down, enough generation demand + reserve
    - Hours of shutdown
    - Less / more than minimum shutdown time
    - Calculate : Cost plant running at minimum
      Cost plant shutting off + start up (banking or cooling)
    - Repeat

Lagrange optimization

- Thesis
- Cost function
- Constraints
    - Generation limits
    - Transmission losses

Chapter 3: Optimal dispatching via nonlinear constraint optimization tools

3.1Economic dispatch

The economic dispatch (ED) problem consists in allocating the total demand among generating units so that the production cost is minimized. Generating units have different production costs depending on the prime energy source used to produce electricity (mainly coal, oil, natural gas, uranium, and water stored in reservoirs). And these costs vary significantly; for example, the marginal costs for nuclear, coal, and gas units may vary considerably, taking on values ranging between $0.03 and $0.20 per kWh. To appreciate the advantages of dispatching a power system according to the solution of the ED problem, consider the case where a power plant supplies 10,000MWduring 1 h at an average cost of $0.05/kWh; if the consumers buy this energy at the rate of $0.06/kWh, this arrangement results in a net profit to the supplier of $100,000/h. In this case an improvement in supply efficiency of just 1% through the use of ED would result in a profit increment of $5000/h or $43.8 million in one year. Also, note that this increment in profit does not necessarily have to end up in its entirety in the pockets of the producer, but, rather, could also be used to reduce the consumer price. Therefore, it is clear that there is a strong incentive for both producers and consumers to increase the efficiency of the generating units

3.2 Lagrangian multiplier and karush kuhn tucker conditions

Definition of the karush kuhn tucker conditions

The Karush-Kuhn-Tucker (KKT) conditions are conditions that the optimal solutions of a broad range of optimization problems should satisfy. For some problems the KKT conditions cannot be meaningfully formulated, and thus they cannot characterize optimal solutions for these problems. Additionally, KKT conditions can be necessary but not sufficient conditions, i.e., solutions meeting them are not necessarily optimal but optimal solutions need to meet them. Also, KKT conditions are first-order conditions, i.e., conditions that are formulated using first derivative vectors and matrices (gradients and Jacobians). To formulate the KKT conditions it is convenient to define the Lagrangian function as the cost functions in the following 4 cases.

Definition of the Lagrangian multiplier

In mathematical optimization, the method of Lagrange multipliers (named after Joseph Louis Lagrange) is a strategy for finding the local maxima and minima of a function subject to equality constraints.

λ^* in function of the power output

As seen in following cases, the λ^* increases linear with the demand if the transmission losses are not considered. When you implement the transmission losses, this function isn't a linear function anymore.

3.3 Cases

3.3.1 Case 1 = OD with equality constraints

I will solve the following problem with 2 and 3 plants, the simple form. This will optimize the load distribution and minimize the cost of operation.

- An electrical power system comprises two (2) generating units with power outputs P1, P2 and hourly cost given by the equations below:

  $C1 = 100 + 20P1 + 0.025P1^2$

  $C2 = 200 + 25P2 + 0.05P2^2$

- An electrical power system comprises three (3) generating units with power outputs P1, P2, P3 and hourly cost given by the equations below:

  $C1 = 1200 + 20P1 + 0.005P1^2$

  $C2 = 1000 + 30P2 + 0.005P2^2$

  $C3 = 1000 + 10P3 + 0.010P3^2$

3.3.1.1 Solved manually

C1 = 1200 + 20 * P1 + 0.005 * P1^2
C2 = 1000 + 30 * P2 + 0.005 * P2^2
C3 = 1000 + 10 * P3 + 0.010 * P3^2
C_tot = 3200 + 20 * P1 + 30 * P2 + 10 * P3 + 0.005 * P1^2 + 0.005 * P2^2 + 0.010 * P3^2
Constraint: P_demand = P1 + P2 + P3 = 4500MW
F(P1,P2,P3) = 3200 + 20*P1 + 30*P2 + 10*P3 + 0.005*P1^2 + 0.005*P2^2 + 0.010 *P3^2 + λ*(4500-P1-P2-P3)
Partial derivatives:
F(P1) = 20 + 0.010 * P1 - X = 0 -> P1 = X - 20 / 0.010
F(P2) = 30 + 0.010 * P2 - X = 0 -> P2 = X - 30 / 0.010
F(P3) = 10 + 0.020 * P3 - X = 0 -> P3 = X - 10 / 0.020
F(X) = 4500 - P1 - P2 - P3 = 0
4500 - (X - 20 / 0.010) - (X - 30 / 0.010) - (X - 10 / 0.020) = 0
4500 = (X - 20 / 0.010) - (X - 30 / 0.010) - (X - 10 / 0.020)
4500 = 5 * X - 110 / 0.020

-> λ_optimal = 40

P1_optimal = 40 - 20 / 0.010 = 2000
P2_optimal = 40 - 30 / 0.010 = 1000
P3_optimal = 40 - 10 / 0.020 = 1500

Applied on 1 demand level

Two Power Plant Example

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	40	23	40	0	0
P2	40	23	-20 = 0	0	0

C1 = 100 + 20 * (40) + 0.025 * $(40)^2$ = 940 €/h
C2 = 200 + 25 * (0) + 0.05 * $(0)^2$ = 0 €/h
Total cost = C1 + C2 = 940 €/h

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	600	41.667	433.33	0	0
P2	600	41.667	166.667	0	0

C1 = 100 + 20 * (433.33) + 0.025 * $(433.33)^2$ = 13460.972 €/h
C2 = 200 + 25 * (166.667) + 0.05 * $(166.667)^2$ = 5755.644 €/h
Total cost = C1 + C2 + C3 = 19216.617 €/h

Three Power Plant Example

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	4500	40	2000	0	0
P2	4500	40	1000	0	0
P3	4500	40	1500	0	0

If we implement these value's into the cost function, we can determine the cost of every power generation plant and the total cost.

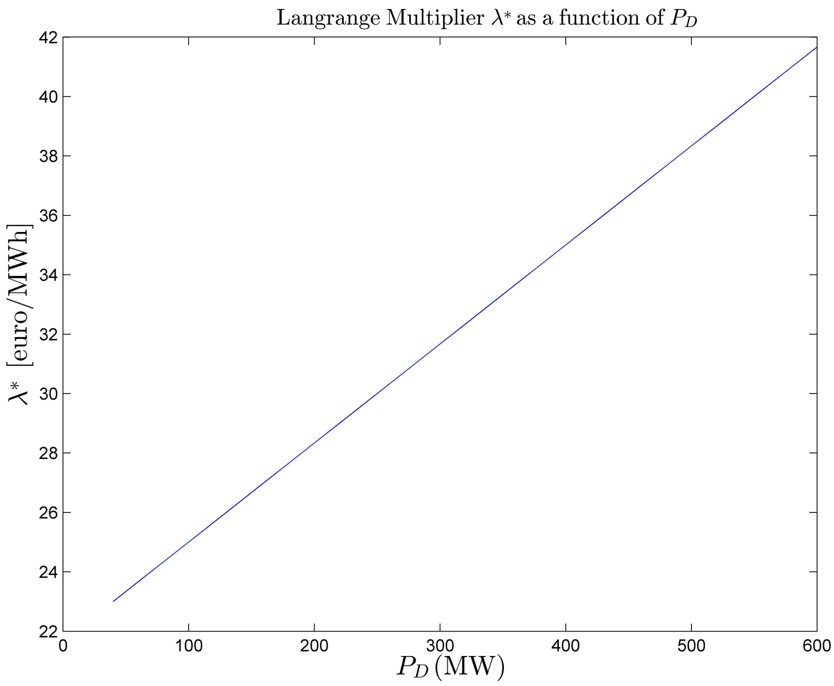
C1 = 1200 + 20 * (2000) + 0.005 * $(2000)^2$ = 61200 €/h
C2 = 1000 + 30 * (1000) + 0.005 * $(1000)^2$ = 36000 €/h
C3 = 1000 + 10 * (1500) + 0.010 * $(1500)^2$ = 38500 €/h
Total cost = C1 + C2 + C3 = 135700 €/h

Applied on more demand levels

Exposito

Now we calculate the λ* for different P_demand levels and plot them onto a graph as seen below. The initial demand level is P_demand = 40 and in steps of 25 goes to P_demand = 600.

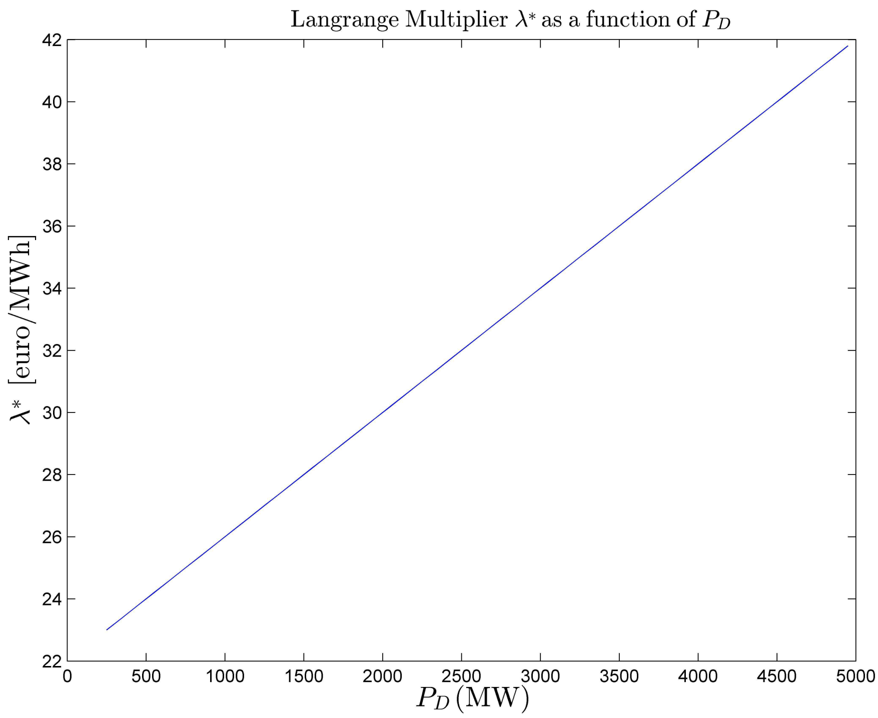
P_demand	λ*
40	23.000
60	23.667
80	24.333
100	25.000
120	25.667
140	26.333
160	27.000
180	27.666
200	28.333
220	29.000
240	30.000
260	30.333
280	31.000
300	31.666
320	32.333
340	33.000
360	33.666
380	34.333
400	35.000
420	35.666
440	36.333
460	37.000
480	37.666
500	38.333
520	39.000
540	39.666
560	40.332
580	40.999
600	41.666



Three Power Plant Example

Now we calculate the λ* for different P_demand levels and plot them onto a graph as seen below. The initial demand level is Pd = 250 and in steps of 100 goes to Pd = 5000.

P_demand	λ*
250	23.000
350	23.400
450	23.800
550	24.200
650	24.600
750	25.000
850	25.400
950	25.800
1050	26.200
1150	26.600
1250	27.000
1350	27.400
1450	27.800
1550	28.200
1650	28.600
1750	29.000
1850	29.400
1950	29.800
...	...
3250	35.000
3350	35.400
3450	35.800
3550	36.200
3650	36.600
3750	37.000
3850	37.400
3950	37.800
4050	38.200
4150	38.600
4250	39.000
4350	39.400
4450	39.800
4550	40.200
4650	40.600
4750	41.000
4850	41.400
4950	41.800
5000	42.000



Observations

Two Power Plant Example

As you can see on the graph, the value of λ* increases linearly with the demand level. This means that the production cost is higher when the demand level raises.

From the multiple values that we calculate, we can say that the participation factors are 2/3 for unit 1 and 1/3 for unit 2. This is a reasonable result as the cost of unit 2 increases with its production at a higher rate than the cost of unit 1, that is, $b1 = 0.05 < b2 = 0.1$. In this sense, we can say that unit 1 is incrementally more efficient than unit 2.

We can deduct from this that unit 1 is more power efficient is than unit 2.

The demand level can be so low that the calculated value of the second generation unit is negative. Using this plant would result in a loss so the plant is shut off or operated at his minimum.

The lower bound λ* is not activated for plant 1 or 2 when applied on the demand level of 40.

This optimal solution satisfies the demand balance equation. However, because the demand is low enough as in this case, the generation output of P2 becomes negative which violates the minimum power output of unit 2. This shows that the power generation limits have not been taking into account in solving the problem, so the results are infeasible.

Three Power Plant Example

As you can see, the value of λ* increases linear with the demand level. This means that the production cost is higher when the demand level raises.

The operational cost of each station is considered only depend on its generated power.

This optimal solution satisfies the demand balance equation. However, because the demand is low enough as in this case, the generation output of P2 becomes negative which violates the minimum power output of unit 2. This shows that the power generation limits have not been taking into account in solving the problem, so the results are infeasible.

3.3.2 Case2 = OD with equality constraints and generation limits

I will solve the following problem with 2 and 3 plants, the simple form and taking the generation limits into account. This will optimize the load distribution and minimize the cost of operation.

- An electrical power system comprises two (2) generating units with power outputs P1, P2 and is constrained by the generation limits. The <u>hourly cost</u> is given by the equations below.

  $C1 = 100 + 20P1 + 0.025P1^2$
  $C2 = 200 + 25P2 + 0.05P2^2$

  <u>Generation limits:</u>

  $0\ MW \leq P1 \leq 400\ MW$
  $0\ MW \leq P2 \leq 300\ MW$

- An electrical power system comprises three (3) generating units with power outputs P1, P2, P3 and is constrained by the generation limits. The <u>hourly cost</u> is given by the equations below.

  $C1 = 1200 + 20P1 + 0.005P1^2$
  $C2 = 1000 + 30P2 + 0.005P2^2$
  $C3 = 1000 + 10P3 + 0.010P3^2$

  <u>Generation limits:</u>

  $1000\ MW \leq P1 \leq 5000\ MW$
  $100\ MW \leq P2 \leq 900\ MW$
  $2000\ MW \leq P3 \leq 3000\ MW$

Two Power Plant Example

There are 2 examples for this case to show the impact of the generation limit on the marginal cost.

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	40	22	40	0	0
P2	40	22	0	3.000	0

$C1 = 100 + 20 * (40) + 0.025 * (40)^2 = 940$ €/h
$C2 = 200 + 25 * (0) + 0.05 * (0)^2 = 0$ €/h
Total cost = C1 + C2 + C3 = 135 €/h

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	600	45	400	0	5.000
P2	600	45	200	0	0

$C1 = 100 + 20 * (400) + 0.025 * (400)^2 = 12100$ €/h
$C2 = 200 + 25 * (200) + 0.05 * (200)^2 = 7200$ €/h
Total cost = C1 + C2 + C3 = 19300 €/h

Three Power Plant Example

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	4500	37.5	1750	0	0
P2	4500	37.5	750	0	0
P3	4500	37.5	2000	12.5	0

$C1 = 1200 + 20 * (1750) + 0.005 * (1750)^2 = 51512.5$ €/h
$C2 = 1000 + 30 * (750) + 0.005 * (750)^2 = 26312.5$ €/h
$C3 = 1000 + 10 * (2000) + 0.010 * (2000)^2 = 61000$ €/h
Total cost = C1 + C2 + C3 = 138825 €/h

Observations

Two Power Plant Example

The solution of the first example implies that unit 2 operates at its minimum power output of 0MW. This is the same as in case 1 because the negative solution of case 1 is equal to 0.

In the second example the upper limit of the first generation plant is reached. This means that the second generation unit has to produce the remaining demanded power even though this plant is less cost effective.

If this result is compared to the first case without generation limits, it becomes clear that the cost increases because the first plant isn't used to its full potential.

The KKT condition is reached, this means that the optimal conditions aren't satisfied.

Three Power Plant Example

The solution implies that the third power generation unit is used at its minimum power output of 2000MW. This is not the same as the first case. This means that the third power plant has to work at a higher power output. The difference in cost/h makes it clear that the third unit is not operating at its optimal marginal cost.

3.2.3 Case 3 = OD with equality constraints and transmission losses

The following problem is solved with 2 and 3 plants, taking the transmission losses into account. This will optimize the load distribution and minimize the cost of operation. The following results will point out that the marginal value λ^* isn't a linear function with the demand. This was an assumption

- An electrical power system comprises two (2) generating units with power outputs P1, P2 and is constrained by the transmission losses. The generation limits are not taken into account. The hourly cost is given by the equations below.

  $C1 = 100 + 20P1 + 0.025P1^2$
  $C2 = 200 + 25P2 + 0.05P2^2$

  Transmission losses:

  $PLOSS(P1,P2) = 0.5P1 + 0.5P2$

  $\alpha = 0.5$, $\beta = 0.5$

- An electrical power system comprises three (3) generating units with power outputs P1, P2, P3 and is constrained by the transmission losses. The generation limits are not taken into account. The hourly cost is given by the equations below.

  $C1 = 1200 + 20P1 + 0.005P1^2$
  $C2 = 1000 + 30P2 + 0.005P2^2$
  $C3 = 1000 + 10P3 + 0.010P3^2$

  Transmission losses:

  $PLOSS(P1,P2) = 0.5P1 + 0.5P2$

  $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 0$

Applied on 1 demand level

Two Power Plant Example

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	40	48.67	86.67	0	0
P2	40	48.67	0	0	0

$C1 = 100 + 20 * (86.67) + 0.025 * (86.67)^2 = 2021.19$ €/h
$C2 = 200 + 25 * (0) + 0.05 * (0)^2 = 0$ €/h
Total cost = C1 + C2 + C3 = 2021.19 €/h

Three Power Plant Example

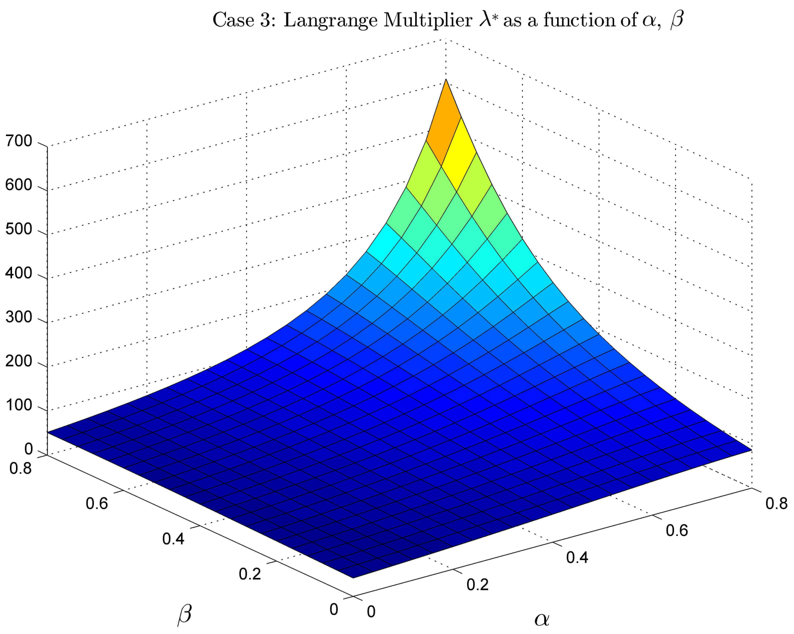
	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	4500	75	1750	0	0
P2	4500	75	750	0	0
P3	4500	75	3250	0	0

$C1 = 1200 + 20 * (1750) + 0.005 * (1750)^2 = 51512.5$ €/h
$C2 = 1000 + 30 * (750) + 0.005 * (750)^2 = 26312.5$ €/h
$C3 = 1000 + 10 * (3250) + 0.010 * (3250)^2 = 139125$ €/h
Total cost = C1 + C2 + C3 = 216950 €/h

Applied on multiple demand levels

Two Power Plant Example

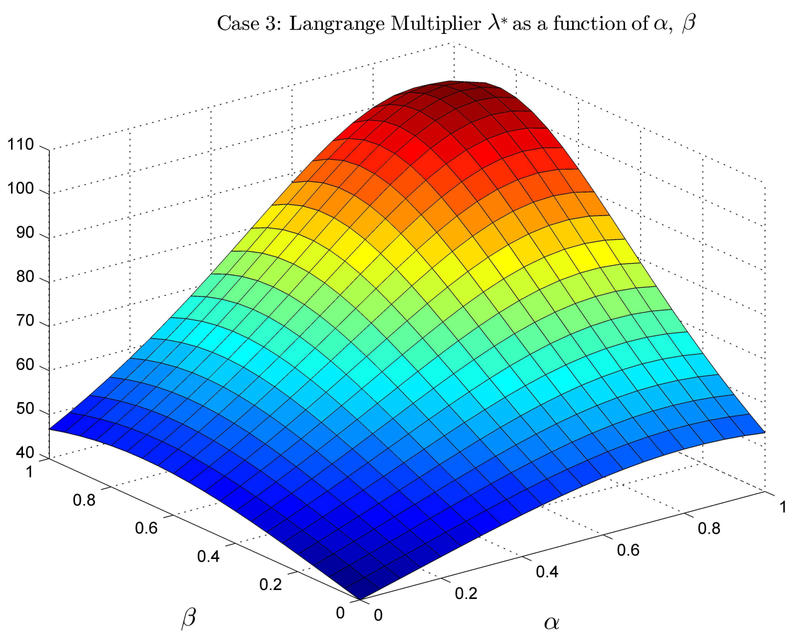
Now we calculate the λ* for different α and β levels and plot them onto a graph as seen on the next page. The initial α and β = 0 and go up to 0.8 in 20 steps. The demand level is fixed on 40MW. The values only go to 0.8 because above that, the marginal cost λ* becomes an infinite high number.



Three Power Plant Example

Now we calculate the λ* for different α and β levels and plot them onto a graph as seen below. The initial α and β = 0 and go up to 1 in 20 steps. The P_demand level is fixed on 4500MW.

The values of α and β are plotted in a 3D graph with α on the X-axis, β on the Y-axis and λ* on the Z-axis.



Observations

Two Power Plant Example

The output value of P1 show that the generated power has to be higher than the demanded power. This is because of the transmission losses. There will always be transmission losses so the output must always be higher.

As seen in the graph, increasing the transmission loss doesn't change the marginal cost when the loss is still low, but when the losses are approaching the value 1, the cost is increases exponentially.

Three Power Plant Example

When interpreting the graph of multiple values of α and β that are plotted in function of the marginal cost function λ*, it is clear that the higher the transmission losses, the marginal cost raises too. This is not a linear function.

Comparison with case 1

In case 1 the calculations aren't affected by the losses. This is a pure theoretical approach to the optimization problem. This is to have a target value of what every generation unit should produce. In a real world optimization problem it's important to factor in the different inequality constraints like the transmission losses. If this is not the case, the delivered power doesn't meet the demanded power. The bigger the losses are, the higher the power output is affected.

3.2.4 Case 4 = OD with equality constraints, generation limits and transmission losses

The following problem is solved with 2 and 3 plants, taking the transmission losses and the generation limits into account. This will optimize the load distribution and minimize the cost of operation.

- An electrical power system comprises two (2) generating units with power outputs P1, P2 and is constrained by the generation limits. The hourly cost is given by the equations below.

  $C1 = 100 + 20P_1 + 0.025P_1^2$
  $C2 = 200 + 25P_2 + 0.05P_2^2$

  Generation limits:

  $0 \text{ MW} \leq P_1 \leq 400 \text{ MW}$
  $0 \text{ MW} \leq P_2 \leq 300 \text{ MW}$

Transmission losses:

$P_{LOSS}(P_1,P_2) = 0.5P_1 + 0.5P_2$
$\alpha = 0.5$, $\beta = 0.5$

- An electrical power system comprises three (3) generating units with power outputs P1, P2, P3 and is constrained by the generation limits. The hourly cost is given by the equations below.

$C1 = 1200 + 20P1 + 0.005P1^2$
$C2 = 1000 + 30P2 + 0.005P2^2$
$C3 = 1000 + 10P3 + 0.010P3^2$

Generation limits:

$1000\ MW \leq P1 \leq 5000\ MW$
$100\ MW \leq P2 \leq 900\ MW$
$2000\ MW \leq P3 \leq 3000\ MW$

Transmission losses:

P_loss(P1,P2) = 0.5P1 + 0.5P2
$\alpha = 0.5$, $\beta = 0.5$, $\gamma = 0$

Two Power Plant Example

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	40	48	80	0	0
P2	40	0	0	0	0

$C1 = 100 + 20 * (80) + 0.025 * (80)^2 = 1860$ €/h
$C2 = 200 + 25 * (0) + 0.05 * (0)^2 = 0$ €/h
Total cost = C1 + C2 + C3 = 1860 €/h

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	600	48	300	0	0
P2	600	0	400	0	0

$C1 = 100 + 20 * (300) + 0.025 * (300)^2 = 8350$ €/h
$C2 = 200 + 25 * (400) + 0.05 * (400)^2 = 18200$ €/h
Total cost = C1 + C2 + C3 = 26550 €/h

Three Power Plant Example

	P_demand	λ*	P_output	Lb λ*	Ub λ*
P1	4500	82	2900	0	0
P2	4500	82	900	0	0
P3	4500	82	3000	0	0

C1 = 1200 + 20 * (2900) + 0.005 * $(2900)^2$ = 101250 €/h
C2 = 1000 + 30 * (900) + 0.005 * $(900)^2$ = 32050 €/h
C3 = 1000 + 10 * (3000) + 0.010 * $(3000)^2$ = 121000 €/h
Total cost = C1 + C2 + C3 = 254300 €/h

Observations

Two Power Plant Example

The output value of P1 show that the generated power has to be higher than the demanded power. This is because of the transmission losses. There will always be transmission losses so the output must always be higher.

Because of the generation limits, the power that the first generation unit has to produce is lower.

Three Power Plant Example

The third generation unit must produce at its maximum output. Because of this, the marginal value and therefore the total production cost is higher.

Chapter 4: Conclusion

We show how different constraints can affect the output of a power generation unit. We provide 12 fairly detailed examples of models optimal dispatching whose results can be replicated by the reader. The first example, which looks at the optimal dispatching with only equality constraints, gives us a basic target value that we can base ourselves on for the next calculations.

The second example, which adds the power generation limits to the first basic example, lets us understand the impact of the limits of different units. This affects all included generating units. Because of this constraint, the most cost effective unit can only be used to its limit and the lesser cost effective units have to produce more. This results in a higher cost thus a higher price.

In the third example, the transmission losses are factored in. This shows that because of these losses, a higher power output is required to meet the power demand. Concluding from the graph we can see that the transmission losses are not a linear function with the marginal cost.

In the fourth example, we factor in the generation limits together with the transmission losses. This is the most realistic approach to optimizing the generation units we calculate in this thesis. It considers two realistic constraints that are represented in practical calculations.

References used

- MATLAB R2014a + Optimization Toolbox (http://www.mathworks.com/products/optimization/)

- the “Kirschen “ TEXTBOOK by Daniel S. Kirschen, Goran Strbac « Fundamentals of Power System Economics (http://eu.wiley.com/WileyCDA/WileyTitle/productCd-0470845724.html)
- Chapters 4 + 5 + 6 from the Kirchen textBook + PPT/PDF presentations, Lectures and Matlab Examples from accompanying site
- the “Exposito” TEXTBOOK by Antonio Gomez-Exposito "Electric Energy Systems: Analysis and Operation" especially chapter 5 (Economics of Electricity Generation)
- Electric Economy Notes (in English) by Prof. G. Pagiatakis ASPETE
- TEXTBOOK Allen J. Wood, Bruce F. Wollenberg «Power Generation, Operation and Control (3rd Ed.)»(http://bookzz.org/book/1057035/57e086)
- the Overbye TEXTBOOK by J. Glover, M. S. Sarma, Thomas Overbye “Power System Analysis and Design, 5th Edition”
- Edwin K.P. Chong and Stanislaw H. Zak, an introduction to optimization 2nd edition (Wiley 2001)

Appendix

B. Fmincon

Objective functions

Two Power Plant Example

```
function f = Exposito51_2Plants_HourlyCosts_objfcn(x) %

C1 = 100 + 20.0*x(1) + 0.5*0.05*x(1)^2 ;
C2 = 200 + 25.0*x(2) + 0.5*0.10*x(2)^2 ;

f = C1 + C2 ; %
end
```

Three Power Plant Example

```
function f = GPag_3Plants_HourlyCosts_objfcn(x) %

C1 = 1200 + 20 * x(1) + 0.005 * x(1)^2;
C2 = 1000 + 30 * x(2) + 0.005 * x(2)^2 ;
C3 = 1000 + 10 * x(3) + 0.010 * x(3)^2 ;

f = C1 + C2 + C3; %
end
```

Case 1:

Applied on 1 demand level

Two Power Plant Example:

```
% Case 1: Equality constraint, no losses
% % ================================================
clear all ; close all; clc
disp(' ***** 16 April 2016 **** '); disp(date)
disp(' Case 1 = only Equality Constraints - No Losses ')
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 = P_Demand + P_Const_Loss ')
P_Demand = 40

A_eq = -[1 1 ] ; B_eq = -P_Demand
%==== a , b
a = []; b = [];
%==== lb , ub
lb = []; ub = [];
x0 = [0 ; 0 ];

[Popt_Case1, objfcn_value, exitflag, output_Case1, lambda] =
fmincon(@Exposito51_2Plants_HourlyCosts_objfcn,x0,a,b,A_eq,B_eq,lb,ub,[] );

disp(' ***** Optimal Values for P1, P2, lambda.eqlin etc... are... ')
P1_opt = Popt_Case1(1)
P2_opt = Popt_Case1(2)

disp(' ***** Output structure is: '); output_Case1
disp(' **** "lambda.eqlin" is Optimal lambda for Linear equalities ');
lambda.
eqlin
disp(' **** "lambda.lower" is Optimal lambda for Lower bounds "lb"
inequalities ');
lambda.lower
```

```
disp(' **** "lambda.upper" is Optimal lambda for Upper bounds "ub"
inequalities ');
lambda.upper
disp(' **** "lambda.ineqlin" is Optimal lambda for Linear inequalities ');
lambda.
Ineqlin
```

Three Power Plant Example

```
% Case 1: Equality constraint, no losses
% % ================================================
clear all ; close all; clc
disp(' ***** 16 April 2016 **** '); disp(date)
disp(' Case 1 = only Equality Constraints - No Losses ')
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 = P_Demand + P_Const_Loss ')
P_Demand = 4500

A_eq = -[1 1 1] ; B_eq = -P_Demand % -(P_Demand + P_Const_Loss) ;
%==== a , b
a = []; b = [];
%==== lb , ub
lb = []; ub = [];
%=== Initial Guess
%x0 = [P1_min; P2_min; P3_min];
x0 = [0 ; 0 ; 0];

[Popt_Case1, objfcn_value, exitflag, output_Pagiatakis_Case1, lambda] =
fmincon
(@GPag_3Plants_HourlyCosts_objfcn, x0, a, b, A_eq, B_eq,lb,ub,[] ); %

disp(' ***** Optimal Values for P1, P2, lambda.eqlin etc... are... ')
P1_opt = Popt_Case1(1)
P2_opt = Popt_Case1(2)
P3_opt = Popt_Case1(3)

disp(' ***** Output structure is: '); output_Pagiatakis_Case1
disp(' **** "lambda.eqlin" is Optimal lambda for Linear equalities ');
lambda.
eqlin
disp(' **** "lambda.lower" is Optimal lambda for Lower bounds "lb"
inequalities ');
lambda.lower
disp(' **** "lambda.upper" is Optimal lambda for Upper bounds "ub"
inequalities ');
lambda.upper
disp(' **** "lambda.ineqlin" is Optimal lambda for Linear inequalities ');
lambda.
ineqlin
%========== END ============
```

Applied on more demand levels

Two Power Plant Example

```
clear all ; close all; clc
disp(' ***** 1 jun 2016 **** '); disp(date)
disp(' Case 1: only Equality Constraints - No Losses ') % Three Power Plant
Example1 = page17
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 = P_Demand + P_Const_Loss ')
A_eq = -[1 1]
%==== a , b
a = []; b = [];
%==== lb , ub
lb = []; ub = [];
%=== Initial Guess
x0 = [0 ; 0 ];

P_Demand_Initial = 40
P_Demand_Final = 600
Load_Step = 20

%====== {P_Demand, i} LOOP
i=1;
for P_Demand = P_Demand_Initial:Load_Step:P_Demand_Final %
%--- Varying Load = Varying P_Demand
P_Demand_save(i)= P_Demand ;
%--- OPTIMIZE with respect to a Varying Load = P_Demand
B_eq = -P_Demand ;
[Popt, objfcn_value, exitflag, output, lambda ] =
fmincon(@Exposito51_2Plants_HourlyCosts_objfcn,x0,a,b,A_eq,B_eq,lb,ub,[] );

Popt_Vector(:,i)= Popt ;
Objfcn_Value_Vector(i)= objfcn_value ;

lambda_Vector(i) = lambda ;
lambda_equality_Vector(i) = lambda.eqlin ;
i=i+1;
end
```

Three Power Plant Example

```
clear all ; close all; clc
disp(' ***** 16 April 2016 **** '); disp(date)
disp(' Example#1: only Equality Constraints - No Losses ') disp(' ***
Equality Constraints Aeq, Beq reflect the instaneous Power Balance *** ' )
disp(' P1 + P2 + P3 = P_Demand + P_Const_Loss ')

A_eq = -[1 1 1] ;
%==== a , b
a = []; b = [];
%==== lb , ub
lb = []; ub = [];
%=== Initial Guess
x0 = [0 ; 0 ; 0 ];
P_Demand_Initial = 250
P_Demand_Final = 5000
Load_Step = 100

i=1;
for P_Demand = P_Demand_Initial:Load_Step:P_Demand_Final %
P_Demand_save(i)= P_Demand ;
B_eq = -P_Demand;

[Popt, objfcn_value, exitflag, output, lambda ] =
fmincon(@GPag_3Plants_HourlyCosts_objfcn, x0, a,b, A_eq, B_eq,lb,ub,[] );

Popt_Vector(:,i)= Popt ;
Objfcn_Value_Vector(i)= objfcn_value ;
lambda_Vector(i) = lambda ;
lambda_equality_Vector(i) = lambda.eqlin ;
i=i+1;
end
```

Case2:

Two Power Plant Example:

```
% Case 2
% ================================================
clear all ; close all; clc
disp(' ***** 05-May-2016 **** '); disp(date)
disp(' Exposito Example 5.3: includes the Generation Limits {Pmin,Pmax}e
given in
Example 5.1 p. 169 + Power-Balance-Equality-Constraints - No Losses ')
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance P1 + P2
+ P3 = P_Demand + P_Const_Loss.... ' )
disp(' ...whereas Pmin,Pmax inequalities are handled via lb,ub ')

P_Demand = 40
A_eq = -[1 1 ] ; B_eq = -P_Demand
a = []; b = [];
lb = [0 ; 0 ]; ub = [400 ; 300];
x0 = [0 ; 0 ];

[Popt_Case2, objfcn_value, exitflag, output_Case2, lambda] =
fmincon(@Exposito51_2Plants_HourlyCosts_objfcn,x0, a,b,A_eq,B_eq,lb,ub,[] );

disp(' ***** Optimal Values for P1, P2, P3, lambda.eqlin etc... are... ')
P1_opt = Popt_Case2(1)
P2_opt = Popt_Case2(2)

disp(' ***** Output structure is: '); output_Case2
disp(' **** "lambda.eqlin" is Optimal lambda for Linear equalities ');
lambda.
eqlin
disp(' **** "lambda.lower" is Optimal lambda for Lower bounds "lb"
inequalities ');
lambda.lower
disp(' **** "lambda.upper" is Optimal lambda for Upper bounds "ub"
inequalities ');
lambda.upper
disp(' **** "lambda.ineqlin" is Optimal lambda for Linear inequalities ');
lambda.
ineqlin
disp(' ***** Optimal Value of OBJ_FUNCTION is: '); objfcn_value
%========== END ============
```

Three Power Plant Example:

```
% Case 2
% % ================================================
clear all ; close all; clc
disp(' ***** 16 April 2016 **** '); disp(date)
disp(' Case 1 = only Equality Constraints - No Losses ')
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 = P_Demand + P_Const_Loss ')

P_Demand = 4500
A_eq = -[1 1 1] ; B_eq = -P_Demand
a = []; b = [];
lb = [1000 ; 100 ; 2000]; ub = [5000 ; 900 ; 3000];
x0 = [0 ; 0 ; 0];
 [Popt_Case2, objfcn_value, exitflag, output_Pagiatakis_Case2, lambda] =
fmincon
(@GPag_3Plants_HourlyCosts_objfcn, x0, a, b, A_eq, B_eq,lb,ub,[] );

disp(' ***** Optimal Values for P1, P2, lambda.eqlin etc... are... ')
P1_opt = Popt_Case2(1)
P2_opt = Popt_Case2(2)
P3_opt = Popt_Case2(3)

disp(' ***** Output structure is: '); output_Pagiatakis_Case2
disp(' **** "lambda.eqlin" is Optimal lambda for Linear equalities ');
lambda.
eqlin
```

```
disp(' **** "lambda.lower" is Optimal lambda for Lower bounds "lb"
inequalities ');
lambda.lower
disp(' **** "lambda.upper" is Optimal lambda for Upper bounds "ub"
inequalities ');
lambda.upper
disp(' **** "lambda.ineqlin" is Optimal lambda for Linear inequalities ');
lambda.
ineqlin
%========== END ============
```

Case 3:

Applied on 1 demand level

Two Power Plant Example

```
clear all ; close all ; clc
disp(' ==== Case 3 = LOSSES + No Constraints - ');
disp(' P_Demand = Constant ')
P_Demand = 40
P1_min = 0; P2_min = 0 ;
P1_max = 300; P2_max = 400 ;
%==== a , b
a = []; b = [];
%==== lb , ub
disp(' do NOT take into account the constraints Pmin Pmax'); lb = []; ub =
[];
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 + P3 = P_Demand + P_Loss WITH P_Loss(P1,P2,P3) = alpha*P1 +
beta*P2 +
gamma*P3')
disp(' hence.... ')

alpha = 0.5
beta = 0.5
a_eq2 = -[1-alpha 1-beta ] ;
b_eq2 = -P_Demand ;

%=== Initial Guess
x0 = [P1_min; P2_min];

[Popt21a, objfunction_Value2, exitflag2, output2, lambda2] =
fmincon(@Exposito51_2Plants_HourlyCosts_objfcn,x0, a, b, a_eq2, b_eq2,lb,ub)

disp(' ***** Optimal Values for Pa, Pb, lambda.eqlin are... ')
P1_opt = Popt21a(1)
P2_opt = Popt21a(2)
disp(' ***** Optimal lambda2 = ')
lambda2.eqlin
disp(' ***** Optimal Value of OBJ FUNCTION is: ')
objfunction_Value2
```

Three Power Plant Example

```
clear all ; close all ; clc
disp(' ==== Case 3 = LOSSES + No Constraints - ');
disp(' P_Demand = Constant ')

P_Demand = 4500
P1_min = 1000; P2_min = 100 ; P3_min = 2000 ;
P1_max = 5000; P2_max = 900 ; P3_max = 3000 ;

%==== a , b
a = []; b = [];
%==== lb , ub
disp(' do NOT take into account the constraints Pmin Pmax'); lb = []; ub =
[];
```

```
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 + P3 = P_Demand + P_Loss WITH P_Loss(P1,P2,P3) = alpha*P1 +
beta*P2 +
gamma*P3')
disp(' hence.... ')

alpha = 0.5
beta = 0.5
gamma = 0
a_eq2 = -[1-alpha 1-beta 1-gamma] ;
b_eq2 = -P_Demand ;

%=== Initial Guess
x0 = [P1_min; P2_min; P3_min];

[Popt21a, objfunction_Value2, exitflag2, output2, lambda2] = fmincon
(@GPag_3Plants_HourlyCosts_objfcn , x0, a, b, a_eq2, b_eq2,lb,ub)

disp(' ***** Optimal Values for Pa, Pb, lambda.eqlin are... ')
P1_opt = Popt21a(1)
P2_opt = Popt21a(2)
P3_opt = Popt21a(3)
disp(' ***** Optimal lambda2 = ')
lambda2.eqlin
disp(' ***** Optimal Value of OBJ FUNCTION is: ')
objfunction_Value2
```

Applied on multiple demand levels

Two Power Plant Example

```
clear all ; close all ; clc
disp(' ==== Case 4 = Linear Transm LOSSES + No Gen Constraints ');
disp(' P_Demand = Constant ')
P_Demand = 40

%==== a , b
a = []; b = [];
%==== lb , ub
lb = []; ub = [];
disp(' Do not take into account the constraints Pmin Pmax');
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 = P_Demand + P_Loss WITH P_Loss(P1,P2,) = alpha*P1 + beta*P2
')
disp(' hence.... ')
%-------- alpha initialization --------
alpha_final = 0.9 ;
alpha_init = 0;
alpha_grid_points = 20 ;
alphastep = (alpha_final - alpha_init)/alpha_grid_points
alpha_save = zeros(alpha_grid_points,1);
%-------- beta initialization --------
beta_final = 0.9 ;
beta_init = 0;
beta_grid_points = 20 ;
betastep = (beta_final - beta_init)/beta_grid_points
beta_save = zeros(beta_grid_points,1);

%----- alpha, i LOOP
i=1;
for alpha = alpha_init:alphastep:alpha_final
alpha_save(i)= alpha ;

i=i+1;
end

%----- beta, j LOOP
j=1;
for beta = beta_init:betastep:beta_final
beta_save(j) = beta ;
j=j+1;
end

for i=1:alpha_grid_points+1
for j=1:beta_grid_points+1
A_eq = -[1-alpha_save(i) 1-beta_save(j)] ;
B_eq = -P_Demand ;
%=== Initial Guess
x0 = [ 0 ; 0];
 [Popt03a, objfunction_Value03a, exitflag03a, output03a, lambda_03a] =
fmincon
(@Exposito51_2Plants_HourlyCosts_objfcn, x0, a, b, A_eq, B_eq, lb,ub,[]) ;
lambda_eqlin_Vector(j, i) = lambda_03a.eqlin ;
end
end

%======== PLOT
fig=10
figure(fig); fig=fig+1;
surf(beta_save, alpha_save, lambda_eqlin_Vector )
xlabel('$\alpha$ ' , 'Interpreter','latex','FontName', 'Times New
Roman','fontsize',
12);
ylabel('$\beta$ ' , 'Interpreter','latex','FontName', 'Times New
Roman','fontsize',
12);
title('Case 3: Langrange Multiplier $\lambda^{*}$ as a function of
\alpha,~\beta$',
'Interpreter','latex','FontName', 'Times New Roman','fontsize',10)
print -djpeg -r600 GPag_3Plants_LinLossAlphaBeta_lambda_jpeg600
```

Three Power Plant Example

```
clear all ; close all ; clc
disp(' Case 3 = Linear Transm LOSSES + No Gen Constraints ');
disp(' P_Demand = Constant ')
P_Demand = 4500
%==== a , b
a = []; b = [];
%==== lb , ub
lb = []; ub = [];
disp('Do not take the constraints Pmin Pmax into account');
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 + P3 = P_Demand + P_Loss WITH P_Loss(P1,P2,P3) = alpha*P1 +
beta*P2 +
gamma*P3')
disp(' hence.... ')
gamma = 0
%-------- alpha initialization --------
alpha_final = 1 ;
alpha_init = 0;
alpha_grid_points = 20 ;
alphastep = (alpha_final - alpha_init)/alpha_grid_points
alpha_save = zeros(alpha_grid_points,1);
%-------- beta initialization --------
beta_final = 1 ;
beta_init = 0;
beta_grid_points = 20 ;
betastep = (beta_final - beta_init)/beta_grid_points
beta_save = zeros(beta_grid_points,1);
%----- alpha, i LOOP
i=1;
for alpha = alpha_init:alphastep:alpha_final
alpha_save(i)= alpha ;
i=i+1;
end
%----- beta, j LOOP
j=1;
for beta = beta_init:betastep:beta_final
beta_save(j) = beta ;
j=j+1;
end % beta, j LOOP stop

for i=1:alpha_grid_points+1 %alpha = 0:0.1:1
for j=1:beta_grid_points+1
A_eq = -[1-alpha_save(i) 1-beta_save(j) 1-gamma] ; % [1-alpha 1-beta 1-
gamma]
B_eq = -P_Demand ; % P_Demand
%=== Initial Guess
x0 = [ 0 ; 0 ; 0 ];


 [Popt03a, objfunction_Value03a, exitflag03a, output03a, lambda_03a] =
fmincon
(@GPag_3Plants_HourlyCosts_objfcn , x0, a, b, A_eq, B_eq, lb,ub,[]) ;
%,optimset
lambda_eqlin_Vector(j, i) = lambda_03a.eqlin ;
end
end
%======== PLOT
fig=10
figure(fig); fig=fig+1;
surf(beta_save, alpha_save, lambda_eqlin_Vector ) %plot(alpha_save,
lambda_eqlin_Vector )
xlabel('$\alpha$ ' , 'Interpreter','latex','FontName', 'Times New
Roman','fontsize',
12);
ylabel('$\beta$ ' , 'Interpreter','latex','FontName', 'Times New
Roman','fontsize',
12);
title('Case 3: Langrange Multiplier $\lambda^{*}$ as a function of
$\alpha,~\beta$',
```

```
'Interpreter','latex','FontName', 'Times New Roman','fontsize',10)
print -djpeg -r600 GPag_3Plants_LinLossAlphaBeta_lambda_jpeg600
```

Case 4

Two Power Plant Example

```
clear all ; close all ; clc
disp(' Case 4 = LOSSES + Generation limits '); % PRESS ANY KEY pause
disp(' P_Demand = Constant ')

P_Demand = 40

P1_min = 0; P2_min = 0 ;
P1_max = 300; P2_max = 400 ;

%==== a , b
a = []; b = [];
%==== lb , ub
lb = [0 ; 0]; ub = [300 ; 400];

disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 + P3 = P_Demand + P_Loss WITH P_Loss(P1,P2,P3) = alpha*P1 +
beta*P2 +
gamma*P3')

disp(' hence.... ')
alpha = 0.5
beta = 0.5

a_eq2 = -[1-alpha 1-beta ] ;
b_eq2 = -P_Demand ;

%=== Initial Guess
x0 = [P1_min; P2_min];

[Popt21a, objfunction_Value2, exitflag2, output2, lambda2] =
fmincon(@Exposito51_2Plants_HourlyCosts_objfcn , x0,a,b,a_eq2, b_eq2,lb,ub)

disp(' ***** Optimal Values for Pa, Pb, lambda.eqlin are... ')
P1_opt = Popt21a(1)
P2_opt = Popt21a(2)

disp(' ***** Optimal lambda2 = ')
lambda2.eqlin
disp(' ***** Optimal Value of OBJ FUNCTION is: ')
objfunction_Value2
```

Three Power Plant Example

```
clear all ; close all ; clc
disp(' Last Touch = 31Jan2016 ')
disp(' Case 4 = LOSSES + Generation limits - ');
disp(' P_Demand = Constant ')
P_Demand = 4500
P1_min = 1000; P2_min = 100 ; P3_min = 2000 ;
P1_max = 5000; P2_max = 900 ; P3_max = 3000 ;
%==== a , b
a = []; b = [];
%==== lb , ub
lb = [1000 ; 100 ; 2000]; ub = [5000 ; 900 ; 3000];
disp(' *** Equality Constraints Aeq, Beq reflect the instaneous Power
Balance *** ' )
disp(' P1 + P2 + P3 = P_Demand + P_Loss WITH P_Loss(P1,P2,P3) = alpha*P1 +
beta*P2 +
gamma*P3')
disp(' hence.... ')
alpha = 0.5
beta = 0.5
gamma = 0
a_eq2 = -[1-alpha 1-beta 1-gamma] ;
b_eq2 = -P_Demand ;
```

```
%=== Initial Guess
x0 = [P1_min; P2_min; P3_min];
%x0 = [ 0 ; 0 ; 0 ];
[Popt21a, objfunction_Value2, exitflag2, output2, lambda2] = fmincon
(@Exam_A21a_3Thermal_withLosses_objfcn , x0, a, b, a_eq2, b_eq2,lb,ub)
disp(' ***** Optimal Values for Pa, Pb, lambda.eqlin are... ')
P1_opt = Popt21a(1)
P2_opt = Popt21a(2)
P3_opt = Popt21a(3)
disp(' ***** Optimal lambda2 = ')
lambda2.eqlin
disp(' ***** Optimal Value of OBJ FUNCTION is: ')
objfunction_Value2
```