Α.Σ.ΠΑΙ.Τ.Ε.

ΑΝΩΤΑΤΗ ΣΧΟΛΗ ΠΑΙΔΑΓΩΓΙΚΗΣ & ΤΕΧΝΟΛΟΓΙΚΗΣ ΕΚΠΑΙΔΕΥΣΗΣ

DESIGN OF "PI" SPEED CONTROLLER FOR DC MOTORS

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KATARZYNA CIURUS

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1. INTRODUCTION

1.2. History about Controller

Negative feedback was first used by James Watt in 1788. In his steam machine he mounted a centrifugal ball regulator with the possibility of proportional control – when the machine is at rest, the weights (balls) are at the bottom and the throttle is maximally open. The rotating wheel of the steam engine is connected to the speed regulator - the balls start to rotate. Two forces act on the bullets - the gravitational force and the centrifugal force. When the balls are raised to a certain height, the throttle is fully closed and the engine's power drops.

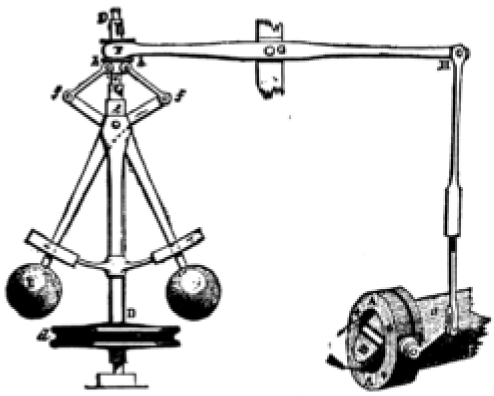


FIG. 4.—*Governor and Throttle-Valve.*

We had to wait around 145 years for the next stage of controller development when in 1933, "Taylor Instrument Company" (today's ABB) introduced the first model of the Full Scope 56R, allowing full tuning of the pneumatic regulator with the possibility of proportional regulation.

The idea was based on forcing the proportional regulator to try to achieve the value of setting so that the real deviation becomes zero when the controller stops working. This was achieved by gradually raising (lowering) the artificial setting, until the real deviation remained non-zero. The automatic operation of changing the setting means that we have to integrate the deviation and add an integral to the input signal of the proportional component of the controller. As a result, we obtain a Proportional-integral (PI) regulator that still generates an increasing output value until the error is eliminated. The first model using the integral effect was Model 40 pneumatic controller introduced to the market in 1934.

During the past two centuries, the focus was on the use of electronic components for controllers (tube technology and semiconductors). In 1976, Rochester Instrument Systems (now part of AMETEK Power Instruments,) launched Media - the first integrated digital implementation of the PI and PID controller.

1.2. Why do we need the PI controller

The PI controller is used in the case of rapid changes of the set point value, so with follow-up control. The PI controller in the steady state reduces the control error to zero. The higher the K_p gain and the shorter the T_i integration time, the faster the regulator operates, while at the same time approaching the stability limit. This is manifested by the tendency to oscillate.

1.3 Summary of the chapters

In chapter two, we will present the general characteristics of the open-loop system and the formulation of the differential equation. We will state the Laplace and state-space methods to find the relation between input (V_a) and output (ω). In the end we will present some examples of a motor working with different load torque levels.

In chapter three, we will present the general characteristic of the individual parts of the PI controller and how they affect the input signal. Subsequently, we will present the design and mode of action of a PI controller. The next step will be to present the work of the PI controller in real time with real motor parameters. We will focus on the analysis of the impact of the response time to changes in the system on the introduction of oscillations to the output signal of the PI controller.

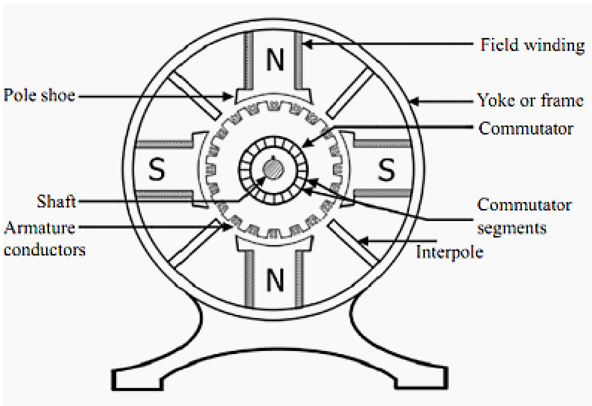
2. Open Loop System

2.1. DC Motors

DC motor is any of a class of rotary electrical machines that converts direct current electrical energy into mechanical energy. The most common types rely on the forces produced by magnetic fields. Nearly all types of DC motors have some internal mechanism, either electromechanical or electronic, to periodically change the direction of current flow in part of the motor.

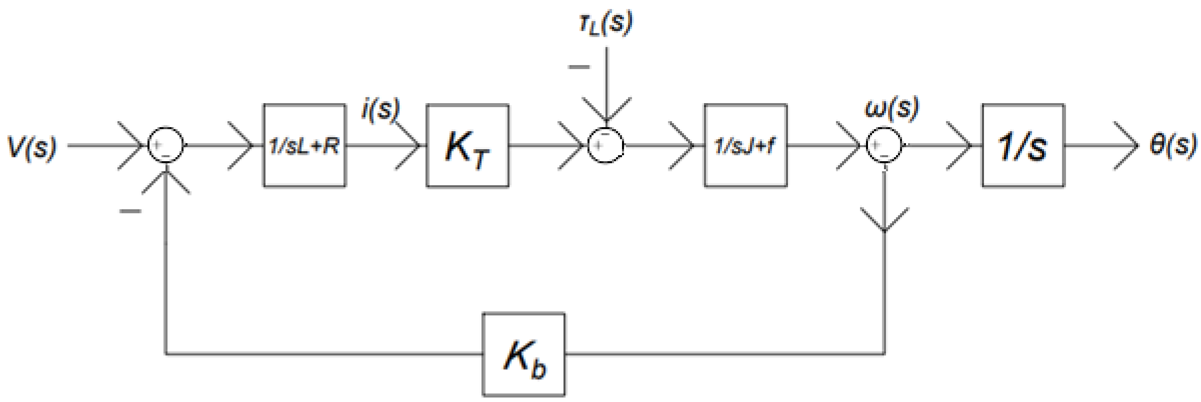
2.1.1. Principle of operation

A coil of wire with a current running through it generates an electromagnetic field aligned with the center of the coil. The direction and magnitude of the magnetic field produced by the coil can be changed with the direction and magnitude of the current flowing through it. Our DC motor has a stationary set of magnets in the stator and an armature with a few windings of insulated wire wrapped around a soft iron core that concentrates the magnetic field. The ends of the wire winding are connected to a commutator. The commutator allows each armature coil to be energized in turn and connects the rotating coils with the external power supply through brushes. (Brushless DC motors have electronics that switch the DC current to each coil on and off and have no brushes.)The total amount of current sent to the coil. The sequence of turning a particular coil on or off dictates what direction the effective electromagnetic fields are pointed. By turning on and off coils in sequence a rotating magnetic field can be created. These rotating magnetic fields interact with the magnetic fields of the magnets (permanent or electromagnets) in the stationary part of the motor (stator) to create a torque on the armature which causes it to rotate.



2.2. Block diagram of the Open Loop System

Power of the input voltage signal V_a is reduced by back electromotive force (K_b is formed in the windings of the armature when the motor starts working) and supplied to the armature. In the armature the magnetic field starts being generated due to the windings inside the rotor. Thus we can see the resistance (R_a) and the inductance (L_a) caused by current flow i_a (current is the output of the armature). Consequently, the motor torque is generated with constant torque (K_T). In order to use this rotating, we connect with motor load torque (τ_L). Final torque is lower than reference torque because of the friction (f) formed in brushes and bearings and the moment of inertia (J) which is constant depends on the mass and radius motors shaft (Newton's law).



2.3. Differential equations

The inductance is a constant. The derivative of the current with the time equal to the resistance value during the current flow $i(t)$, generated by input voltage V(t) and back electromotive force depends only on angular speed (because the number of stator turns and magnetic field produced by rotor magnets are invariable in DC motor)

$$\boldsymbol{L\frac{di}{dt} = -Ri(t) - K_b\omega(t) + V(t)}$$

$K_b\omega(t)$ – back electromotive force

$L\frac{di}{dt}$ – change of current

$V(t)$ – voltage source

The moment of inertia (J) and coefficiency of motor torque are constant (K_T depends on magnetic force which invariable), it means that the torque is only affected by the current. Friction is proportional to the angular speed (formed in brushes and bearings) and the load moment remains unchanged.

$$\boldsymbol{J\frac{d\omega}{dt} = \tau_m - \tau_L - f\omega_R = -f\omega(t) + K_T i(t) - \tau_L(t)}$$

$J\frac{d\omega}{dt}$ – moment of inertia

$\tau_L(t)$ – load torque

$\tau_m(t)$ =$K_T i(t)$ – motor torque

$f\omega(t)$ – friction torque

K_T – torque constant

2.4. Transfer function – Laplace

An easy way to control the motor is to change the current because the motor torque is dependent on current value ($\tau_m = K_T i$). Taking this into account we create an inner current control loop which is controlled by voltage (input signal). In order to calculate the current and the angular speed we should convert the equations 2.3.1.and 2.3.2.to transfer functions with zero initial conditions:

$$\boldsymbol{L\frac{di}{dt} = -Ri(t) - K_b\Omega(t) + V(t)} \qquad \textbf{2.3.1.}$$

$$\boldsymbol{sLI(s) = -RI(s) - K_b\Omega(s) + V(s)}$$

$$\boldsymbol{sLI(s) + RI(s) = -K_b\Omega(s) + V(s)}$$

$$\boldsymbol{I(s)(sL + R) = -K_b\Omega(s) + V(s)}$$

$$\boldsymbol{I(s) = \frac{-K_b\Omega(s) + V(s)}{(sL + R)} = \frac{1}{(sL + R)} \cdot -K_b\Omega(s) + V(s)}$$

$$\boldsymbol{J\frac{d\omega}{dt} = -f\Omega(t) + K_T I(t) - \tau_L(t)} \qquad \textbf{2.3.2.}$$

$$\boldsymbol{sJ\Omega(s) = -f\Omega(s) + K_T I(s) - T_L(s)}$$

$$\boldsymbol{sL\Omega(s) + f\Omega(s) = K_T \mathrm{I}(s) - T_L(s)}$$

$$\boldsymbol{\Omega(s)(sL + f) = K_T I(s) - T_L(s)}$$

$$\boldsymbol{\Omega(s) = \frac{K_T I(s) - T_L(s)}{(sL + f)} = \frac{1}{(sL + f)} \cdot K_T I(s) - T_L(s)}$$

2.5. State space model

2.5.1. The matrices A,B,C,D

Linear dynamic stationary systems can be described using dependencies input-output (transmittance), using n differential equations of first order, which can be combined into vector-matrix differential equations (state and output equations). The use of vector-matrix notation simplifies the mathematical description of modeled dynamic systems.

$$\dot{\boldsymbol{x}} = \frac{\boldsymbol{dx}}{\boldsymbol{dt}} = \boldsymbol{f}(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{u}(\boldsymbol{t}), \boldsymbol{t})$$

x(t) – vector of dependent variables, vector with dimensions of nx1 and components x1(t),x2(t),…,xn(t)

u(t) – vector exclusions output, control vector with dimension px1 and components u1(t),u2(t),…,un(t)

f – is a (possibly nonlinear) function producing the time derivative (rate of change) of the state vector,

$\frac{dx}{dt}$ - time derivative of the state vector (instant of time)

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$$

$$\boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du}$$

System matrix A - state matrix (basic, fundamental), describes the dynamics of the system control, a matrix of n × n,

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & -\boldsymbol{a_n} \\ \mathbf{1} & \mathbf{0} & \dots & \mathbf{0} & -\boldsymbol{a_{n-1}} \\ \mathbf{0} & \mathbf{1} & \dots & \mathbf{0} & -\boldsymbol{a_{n-2}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{1} & -\boldsymbol{a_1} \end{bmatrix};$$

Input matrix B - input matrix (control), describes the influence of control on the control system, the matrix of dimensions n × p,

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{b_n} & -\boldsymbol{a_n b_0} \\ \boldsymbol{b_{n-1}} & \boldsymbol{a_{n-1} b_0} \\ \boldsymbol{b_1} & -\boldsymbol{a_1 b_0} \end{bmatrix};$$

Matrix C - the output matrix (response), describes how the state variables are transformed into variable outputs, a matrix of q × n,

$$\boldsymbol{C} = [\mathbf{0} \quad \mathbf{0} \quad \cdots \quad \mathbf{0} \quad \mathbf{1}];$$

Matrix D - direct coupling matrix (transmission), matrix with dimensions q × p, $\boldsymbol{D} = \boldsymbol{0}$ when the output doesn't depend on input (only important is state variables)

$$\boldsymbol{D} = \boldsymbol{b_0};$$

2.5.2. The specific A,B,C,D matrices used for the speed control of DC motor

$$\boldsymbol{\frac{d}{dt}\begin{bmatrix}\dot{\theta} \\ i\end{bmatrix} = \begin{bmatrix}-\frac{b}{J} & \frac{K}{J} \\ -\frac{K}{L} & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}\dot{\theta} \\ i\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{L}\end{bmatrix}V}$$

$$\boldsymbol{y = [1 \;\; 0]\begin{bmatrix}\dot{\theta} \\ i\end{bmatrix}}$$

2.5.2.1. A,B,C,D matrices

$$\boldsymbol{A = \begin{bmatrix}0 & 1 & 0 \\ 0 & -\frac{b}{J} & \frac{K}{J} \\ 0 & -\frac{K}{L} & -\frac{R}{L}\end{bmatrix};}$$

$$\boldsymbol{B = \begin{bmatrix}0 & 0 & \frac{1}{L}\end{bmatrix};}$$

$$\boldsymbol{C = [1 \quad 0 \quad 0];}$$

$$\boldsymbol{D = 0;}$$

2.6. Simulation results based on real motor data

Motor

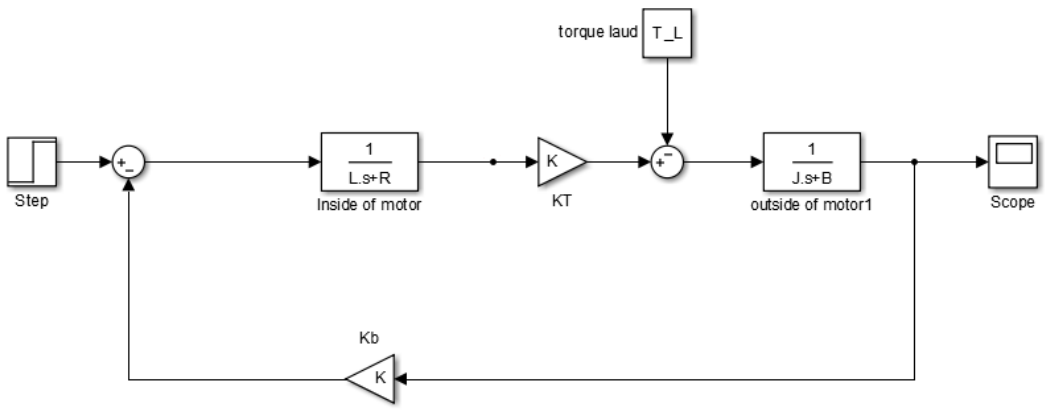
J =0.45 $\frac{kg}{m^2}$;

B = 0.01;

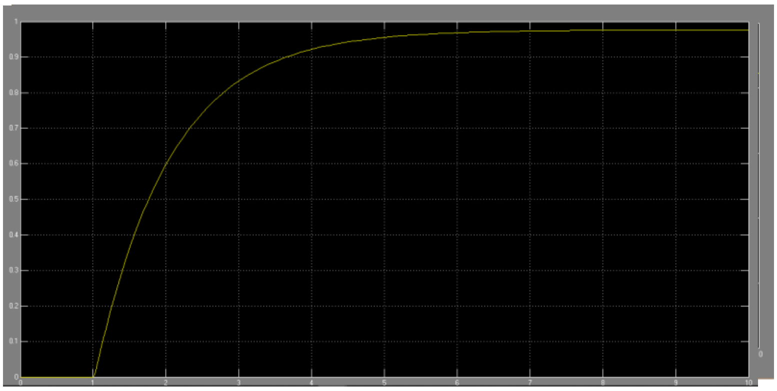
L=0.05 H;

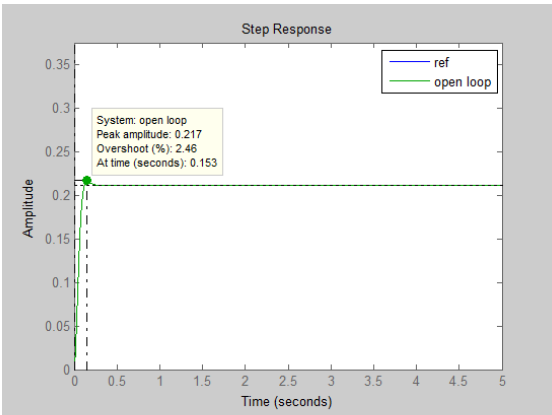
R=2.4 Ω;

K=4.72;



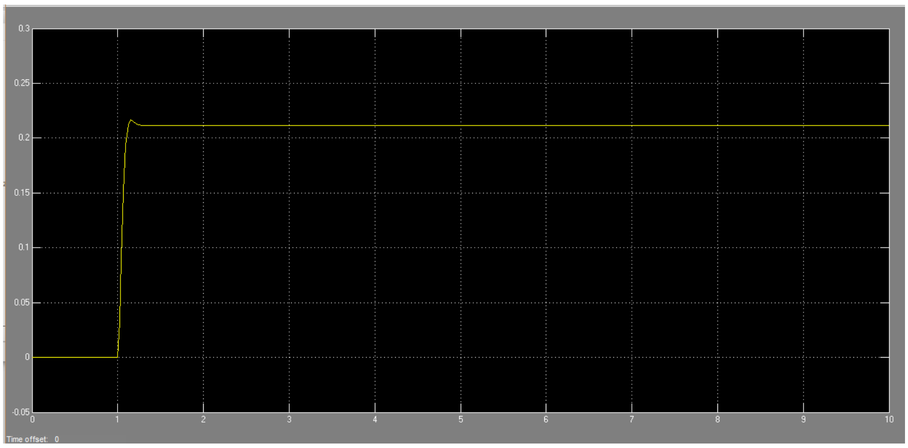
a) T_L=Ø



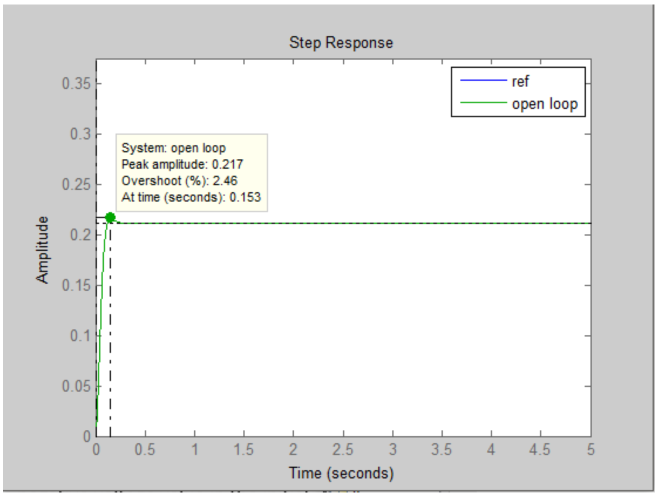


At the chart we can see how the system behaves in an open loop system without load. We have given an input signal of 1V. The response peak was 0.217 volts, this is due to a large Back Electromotive force operating in an engine that is equal to Kb in the system (Kt=Kb=K) – the motor was moving at high angular speed. With the decrease of the Back emf (reduction of the angular speed) value in the program with the Kb value constant, the signal approached 1 V but even with zero Back emf (stop the engine) did not reach the given value - other losses in the system (in fact back emf factor is a constant value dependent on the parts from which the engine is built). Also, the rise time depends on the back emf value. When it is high the rise time decreases, with the back emf 4.72 the rise time was around 0.153 seconds . The steady state reached 0.3 s but the signal was gently oscillating and achieved value 0.212 V.

b) T_L = 0.0025 N



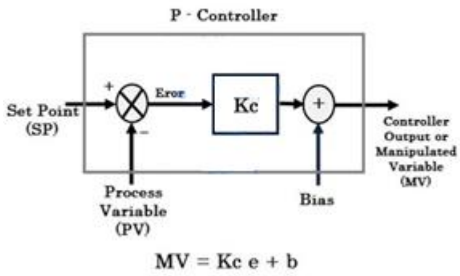
When the system is switched on with a load greater than 0.0025 N, the diagram shows a negative value of voltage which does not occur under normal conditions (the motor does not work). This is due to the characteristics of the engine with its low power. The small load does not introduce change in system. Reducing the back emf (reduce the angular speed of the engine) , only improves the final value of steady state and does not affect the increase of the engine load.



3.Close Loop System

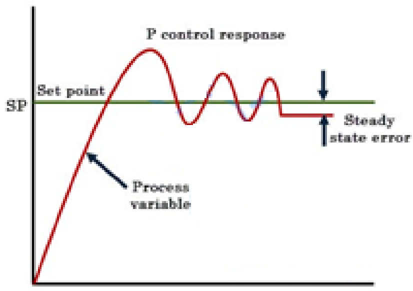
3.1. What is the PI controller ?

a) P-Control Response



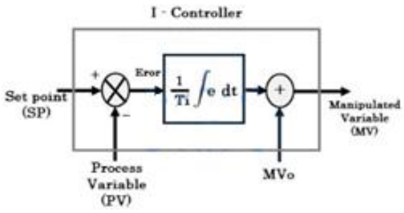
Proportional control or simply P-controller produces the control output proportional to the current error. Here the error is the difference between the set point and process variable (i.e., e = SP – PV). This error value multiplied by the proportional gain (K_c) determines the output response, or in other words proportional gain decides the ratio of proportional output response to error value.

For example, if the magnitude of the error is 20 and K is 4 then the proportional response will be 80. If the error value is zero, controller output or response will be zero. The speed of the response (transient response) is increased by increasing the value of proportional gain K_c. However, if K_c is increased beyond the normal range, process variable starts oscillating at a higher rate and it will cause instability in the system.



Although the P-controller provides stability for the process variable with good speed of response, there will always be an error between the set point and the actual process variable. In most cases, this controller is provided with manual reset or biasing in order to reduce the error when used alone. However, zero error state cannot be achieved by this controller. Hence there will always be a steady state error in the p-controller response as shown in the figure.

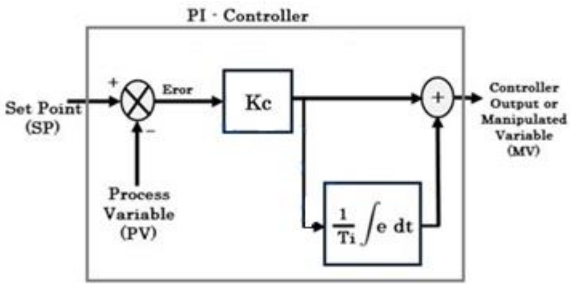
b) I-Control Response



$$MV = \frac{1}{Ti}\int e\,dt + MVo$$

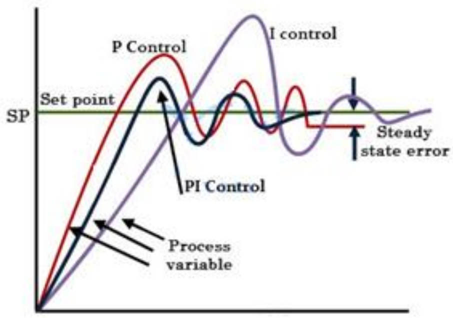
Integral controller or I-controller is mainly used to reduce the steady state error of the system. The integral component integrates the error term over a period of time until the error becomes zero. This means that even a small error value will cause a high integral response. At the zero error condition, it holds the output to the final control device at its last value in order to maintain a zero steady state error, but in case of P-controller, the output is zero when the error is zero.

If the error is negative, the integral response or output will be decreased. The speed of response is slow (means responds slowly) when I-controller alone is used, but improves the steady state response. By decreasing the integral gain Ki, the speed of the response is increased.



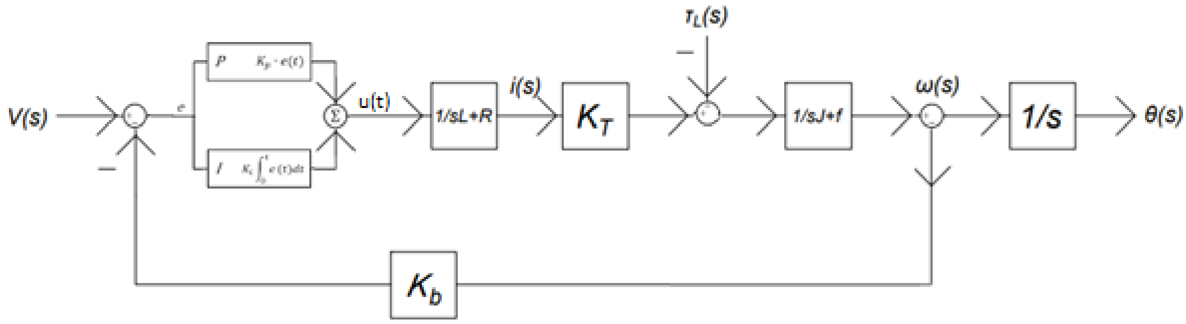
$$MV = Kc\,e + \frac{1}{Ti}\int e\,dt$$

For many applications, proportional and integral controllers are combined to achieve a good speed of response (in case of P controller) and better steady state response (in case of I controller). Most often PI controllers are used in industrial operation in order to improve transient as well as steady state responses. The responses of I-control only, p-control only and PI control are shown in below figure.



3.2. Design of the PI controller

The power of the input voltage signal V_a is reduced by back electromotive force (K_b formed in the windings of the armature when motor starts working). The PI controller gets the input parameter from the motor which is referred as an actual process variable (e). It also requires the desired output, which is referred as set variable, and then it calculates and combines the proportional and integral responses to compute the output. The deviation of the actual value from the desired value in the PI control algorithm causes it to produce the voltage output (u(t)) to the motor depending on the combination of proportional and integral responses. So the PI controller continuously varies the output to the motor till the process variable settles down to the set value. Next, the signal is supplied to the armature. In the armature, the magnetic field is being generated due to the windings inside the rotor. Thus we can see the resistance (R_a) and the inductance (L_a) caused by the current flow i_a (current is the output of the armature). Consequently, the motor torque is created with constant torque (K_T). In order to use this rotating, we make a connection with the motor load torque (τ_L). The final torque is lower than the reference torque because of the friction (f) formed in the brushes and bearings and moment of inertia (J) which is constant depending on the mass and radius motors shaft (Newton's law).



3.3. Simulation results based on real motor data

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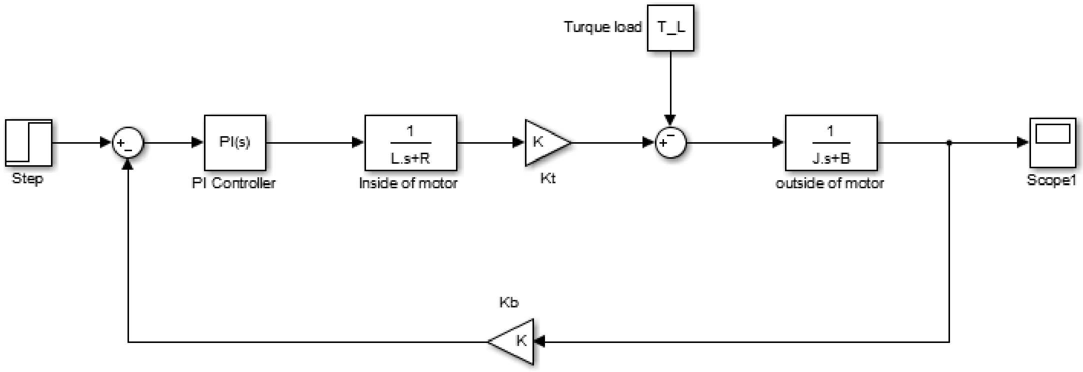
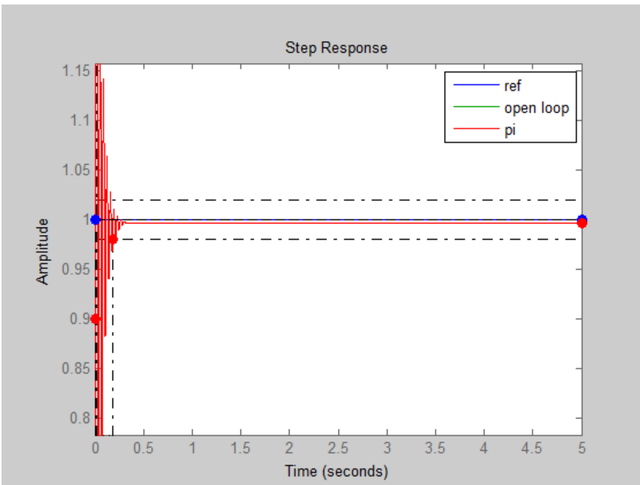
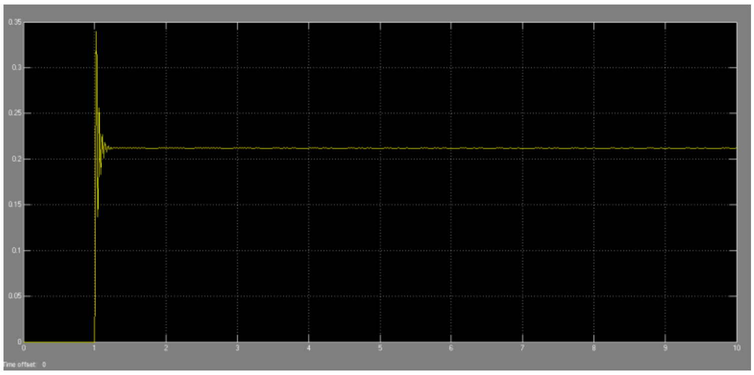
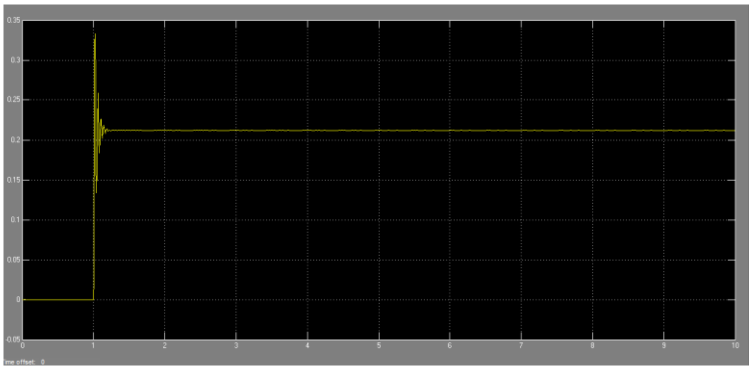


3.3.1. T_L=∅ and T_L = 0.0025 N ; rise time under 0.007 s

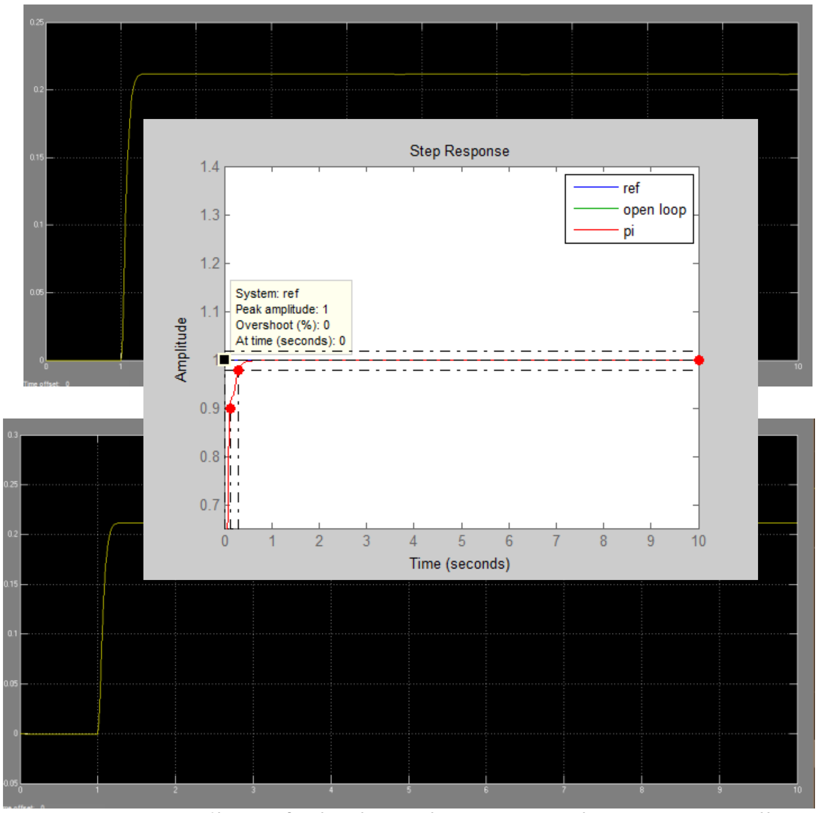






As previously explained, when we want to obtain a fast rising time, similar to a rectangular characteristic, a strong overshoot is created due to the proportional element in the PI controller and its K_c factor. We can compare it to a football player wanting to score a goal. The football player must kick the ball that moves in a straight line so that it hits a point (a set value). If his kick is too strong (large K_c), the ball will hit the point and the ball will go above the goal on the other side (value of the signal goes down), and then the player will shoot again from the other side (value of the signal go up) and so the ball will go close to the goal but will never score a goal (hence the chart oscillations are visible regardless of whether the system is loaded or not the bigger the higher the K_c).

3.3.2. $T_L = \emptyset$ and $T_L =$ 0.0025 N rise time 0.112 s



When we increase the rise time, the I-element of the regulator smoothes the overshoots and reduces the offset and sets the steady state on one level without any oscillations as it did in the open loop. The rise time has 0.112 s but the peak response is equal to the value in steady state which is important from the point of view of proper engine operation. When we load the engine, we have a similar situation to that in the open loop. The engine, regardless of whether it has a PI regulator or not, will not work with a load greater than 0.0025 N due to its output power. The most important advantage of the PI regulator is that we have achieved a rectangular characteristic that was our goal. As you can see, PI regulators work well in systems where the reaction time is longer. They improve the work of the system but do not eliminate the static error.

4. Conclusion

Although the PI controller has many disadvantages, e.g. the inability to predict the changes in velocity, it is a very good device in the system that can be used in many industry brunches. For many systems, there is no need to predict states in the future and its big advantage is a relatively lower price than the PID controller. I only had half a year to explore the PI controller problem, if I had more time for research, I would present a more detailed problem.